Usefulness of the magnetic « corkscrew » for particles beams

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Patrick Lindecker
Maisons-Alfort (France)
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Abstract

This device is mainly used to transform the particles beam axial speed in radial speed. But it has also the capacity, according to the patent, to reversely transform the radial speed in axial speed. This would come to focalize the particles of a beam, so as to make them almost monoenergetic with a small radial speed. This paper studies this device and its possibilities for ions beams. The space charge is not taken into account: either it is very weak or the ions beam is neutralized by electrons.

1) Introduction

For a description of this device, see references [1] and [2] (page 269).

1.1 Principle

This device (called "Charged particle angular momentum changer" in the reference [1]) can permit (according to the patent) to reduce the radial speed of resonant particles and to reduce the dispersion around the nominal speed. It is selective, i.e. it can be used for a given particle (D+ ion or T+ ion, for example) at a certain speed. Ideally, it will act on the particle targeted and will not modify the distribution of other particles (not resonant).

Let's suppose an ion rotating due to its cyclotronic movement:

- with a pulsation w=q .Ba / m with Ba the axial magnetic field,
- on an orbit of Larmor radius Rl = (m . Vr) / (q . Ba), with Vr the radial speed.

The principle to reduce this radial speed Vr is the following. A rotating radial field Br is built by a helix, field supposed in phase with the ion trajectory (in a transverse plane). This radial field Br acting on the axial speed Va, causes a force **F** opposed to the cyclotronic rotation: **F** = q . (**Va** ^ **Br**), with ^ the cross product. Vectors are in bold font. The algebraic value of F is equal to F = q . Va . Br because Va is supposed always perpendicular to Br. See the diagram (a) of the reference [1].

Reversely, if the difference of phases is equal to π, the force F tends to accelerate the cyclotronic rotation.
More generally, with a difference of phases equal to 0 +/- π/2 the rotation is slowed down but with π +/- π/2, the rotation is accelerated.

Of course, in the case of an ion not having any radial speed (Vr=0), this ion is going to acquire radial speed, due to the force F, in any cases.

See the figure 1 below with the ideal case where a particle is focalized.

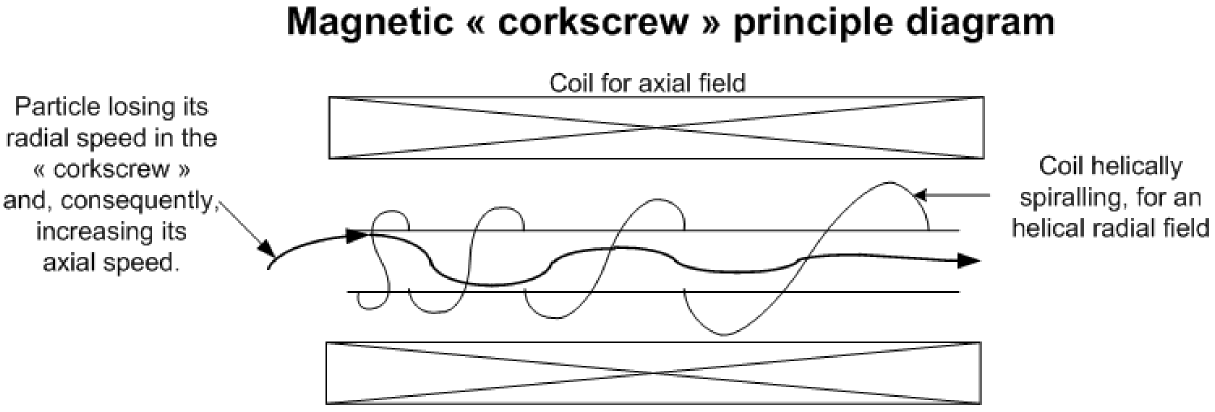


Figure 1

The pitch p (i.e. distance between two turns) of the corkscrew must evolve as: $p(z) = \frac{2\times\pi\times m\times Va(z)}{q\times Ba}$ (resonance condition). Va(z) is the axial speed (the axis being carried by the vector z). Va(z) is not known a priori. It will be necessary either to make a simplified calculation to have an idea of its evolution, or to make a hypothesis on its evolution (preferred by the author). Ba is the confinement axial magnetic field.

<u>1.2 Different types of corkscrews</u>

The most common is a simple helix as shown in the figure 1 above.

Another type is a bifilar winding with currents in opposite direction. See the cut view on figure 2 at the next page (cut of the helix at mid-distance, looking to the exit). The mean axial magnetic field will be removed due to opposite axial components given by each winding. However, locally axial field could be different from 0. This also applies to the quadrifilar winding.

More interesting is the quadrifilar winding shown in figure 2 at the next page. See also references [1] and [4]. Due to symmetries, the magnetic field is much more regular in a section than with a sole winding or, even, a bifilar winding, especially at entrance and exit..

From this quadrifilar winding, it is not necessary to have the complete turns. It can remain, for example, a quarter part of the turn, from -45° to 45° for example. This will permit to localize the magnetic field inside a certain part of the pipe, the quarter of tube between -45° and 45 ° in the example. This will be useful if several beams (4 in the example) are circulating in parallel in the pipe. See figure 2 at the next page. The “-“ wires can be connected in parallel. The “+” wires can be connected in the same way but with a reverse direction of current compared to the “-“ wires. Other connections are possible.

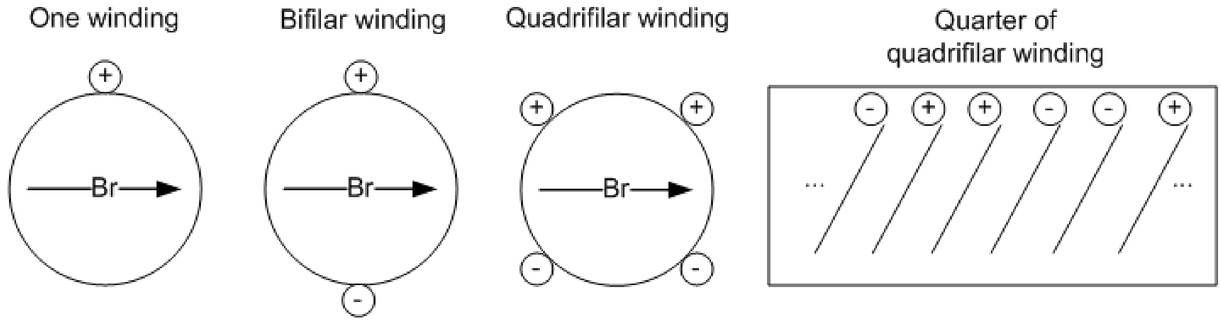


Figure 2

2) Magnetic radial and axial fields in a helix of constant pitch

The radial field is given in the different papers on the subject (reference [1] page 4, for example) by formulas which consider only the field very close to the axis. Afterwards theoretical formulas suppose implicitly that the radial field does not depend on the ion radial position, which is about true close to the axis but wrong beyond. For example, if the radial field Br is supposed equal to 1 T on the axis (r=0) and the helix radius is equal to a, the mean radial field on a section evolves such as described below (for a specific helix):

- At r/a = 0, Br = 1 T
- At r/a = 0.2, Br = 1.18 T
- At r/a = 0.5, Br = 1.93 T
- At r/a = 0.7, Br = 3.18 T
- At r/a = 0.9, Br = 9.17 T

Moreover, on a section, the radial field is bigger close to the helix than far from the helix (which is predictable).

As the radial field depends on r, it is difficult (if not impossible) to determine analytically the ion trajectory.

There are two ways to determine, numerically, the almost exact magnetic field, using, in both cases, the Biot-Savart law:

- either a direct calculation,
- or by determining the exact formulas for a helix and calculating the fields according to these formulas. These ones are given afterwards.

These formulas, in SI, give Bx, By, Bz at a given point P determined by Cartesian coordinates Xp, Yp, Zp. The helix axis is supposed to spread towards the positive z axis. “p” is the pitch (in m), “a” the helix radius (in m), “I” the current through the helix (in A). K = 1 for a helix rotating counterclockwise (for ions) and K = -1 for a helix rotating clockwise (for electrons). “n” is the number of turns.
The initial helix position I is supposed located at any Zi and at any angle θi (i.e. between –π to π) such that Xi=a . cos(θi) and Yi=a . sin(θi). The point M follows the helix. μ0=4.π.1E-7 H/m.

Note that, here, the helix is supposed to develop towards the positive Z axis, so with a positive pitch (p). If the helix would develop towards the negative Z, the pitch would be negative in the formulas.

These integrals are calculated numerically.

$$Bx = \frac{\mu 0 \times \mathrm{I}}{4 \times \pi} \times \int_{\theta i}^{-2 \times K \times n \times \pi + \theta i} \frac{wx \times d\theta}{|PM|^3}$$
$$By = \frac{\mu 0 \times \mathrm{I}}{4 \times \pi} \times \int_{\theta i}^{-2 \times K \times n \times \pi + \theta i} \frac{wy \times d\theta}{|PM|^3}$$
$$Bz = \frac{\mu 0 \times \mathrm{I}}{4 \times \pi} \times \int_{\theta i}^{-2 \times K \times n \times \pi + \theta i} \frac{wz \times d\theta}{|PM|^3}$$

wx=-K.a.cos(θ).(Zp-(Zi-K.p.(θ-θi)/(2.π)) - p/(2.π).(Yp-a.sin(θ))
wy= p/(2.π).(Xp-a.cos(θ)) - K.a.sin(θ). (Zp-(Zi-K.p.(θ-θi)/(2.π))
wz=K.a.sin(θ).(Yp-a.sin(θ)) + K.a.cos(θ).(Xp+K.a.sin(θ))

$|PM| = ((Xp-a.\cos(\theta))^2 + (Yp-a.\sin(\theta))^2 + (Zp-(Zi-K.p.(\theta-\theta i)/(2.\pi))^2)^{0.5}$

After having determinated the exact magnetic field, it is proposed empirical formulas (in cylindrical coordinates) which give approximately the magnetic fields inside an helix of constant pitch, in a more simplified way.
With “Br” for radial field (in T), “Ba” for axial field (in T). ^ means “power” and I0 is the first kind of modified Bessel function. For a given radius, “Br” is, vectorially, approximately constant, whatever is the azimuth, across the considered section.

Ba = 1.22 E-6 . I / p
This axial field must be negligible in front of the main axial field. It can be removed with a bifilar or a quadrifilar winding (see §1.2).

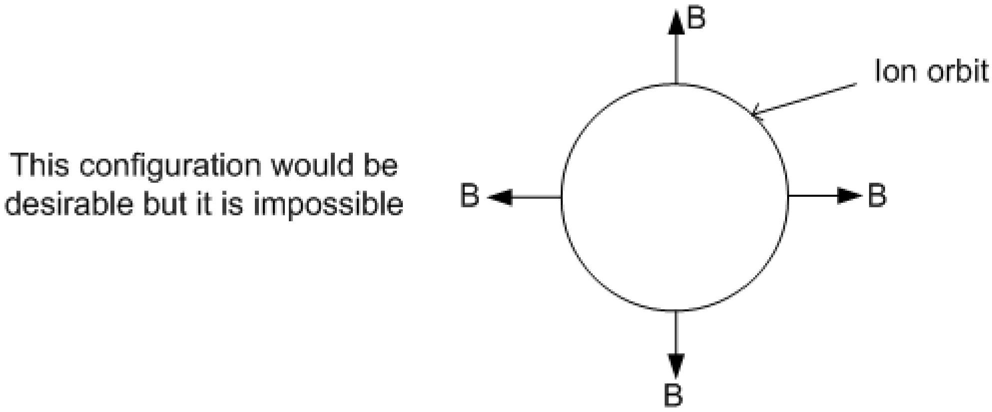
About Br, this formula is worth for p/a between 1.5 and 3 and, with less accuracy, between 1 and 4 (but not beyond). 12 turns are taken into account but Br does not vary much with the number of turns. r/a must be limited to 0.9.

Br_on_the_axis = 4.6 E-8 . I / a . (p/a). sin(0.52 .p / a).
Note that sin(0.52 . p / a) is a correction without any physical meaning.

Br_at_r = Br_on_the_axis . (I0(r/a)) ^ (16 / (p/a))
The correction (I0(r/a)) ^ (16 / (p/a)) has no physical meaning.

The radial field phase follows the helix phase with a delay of 90°.

Note: it is reminded that the flow of magnetic field through a closed surface is nil (from div **B**=0, according to the Maxwell Thomson formula). This means that it is not possible to physically create a magnetic field which would be directed towards the exterior all along an orbit, so as to permanently slow down the ion.



3) Magnetic radial and axial fields in a helix of variable pitch

For this type of helix the most simple is to only use the Biot-Savart law, by breaking down the helix in small elements and calculating the small fields (along x, y, z) generated by each element, the final fields being the sum of these small fields.

However, with a helix of that type, it must be, first, determined the law of pitch evolution (p(z)) along the z axis. We suppose, to simplify, that the helix begins at z=0 and ends at z=L (L being the helix length).

This type of helix is, all cases, better than helixes of constant pitch, but above all for long ones. So it will be implicitly considered long helixes here (see §4.3).

3.1 Pitch law with constant dVr / dz

The reference [3] gives this formula: dVr / dz = - wr . cos(χ) with wr= q.Br/m and χ the angle between the field direction and the ion angle, the ion position being in cylindrical coordinates.
This formula can be deduced from the mean force **F** = q . (**Va** ^ **Br**) (cf. §1), with Va the mean axial speed along the helix. This force projected on the ion trajectory gives, if ABS(χ)<90°, a force Fi opposed to the cyclotronic movement:
|Fi| = q . Va . Br . cos(χ). So dVr / dt = -|Fi| / m = - q . Va . Br . cos(χ) / m
Now dz=Va . dt so dt = dz / Va and Va . dVr / dz = - q . Va . Br . cos(χ) / m which simplifies to dVr / dz = - q . Br . cos(χ) / m.

If we suppose Br approximately constant, the dVr / dz is also constant. If the goal of the helix is to transform all the initial radial speed Vri in axial speed along the helix length “L” then dVr / dz = Vri / L and Vr(z) = Vri + (dVr / dz) . z
As Va(z) = $\sqrt{(Vion^2 - Vr(z)^2)}$ is now known, $p(z) = \frac{2\times\pi\times m\times Va(z)}{q\times Ba}$ is also known at any z.
Vion is the ion speed which can be decomposed in Va (along the z axis) and Vr (in the section perpendicular to z). Note that Vr is, in fact, azimuthal along the circular ion trajectory.

3.2 Pitch law with constant dVa / dz

A more simple law is to consider a linear evolution of p(z). It is enough to suppose that dVa / dz is constant to consequently have p(z) linear between:

- the initial pitch $pi = \frac{2\times\pi\times m\times Vai}{q\times Ba}$, with Vai the initial axial speed equal to Vai = $\sqrt{}$(Vion^2 – Vri^2)
- the final pitch $pf = \frac{2\times\pi\times m\times Vion}{q\times Ba}$

So p(z) = pi + (pf-pi) * z/L

3.3 Comparison between these two types of helix and selection of the best one

According to several simulations done at different intensities, it seems that the helix with a constant dVa / dz pitch law is better in terms of mean radial speed reduction than the same helix with a constant dVr / dz pitch law.
So from now on, it will be only considered helix with constant dVa / dz pitch law.

4) Simulation of the magnetic "corkscrew" and results

4.0 Generalities

The way to calculate the radial field Br and its phase has been introduced in the author's simulator (called "Multiplasma"). The magnitude of the radial field can be adjusted through the current (in A) circulating through the helix.

All tests of §4.1 and 4.2 are done on an ion injected at 1.932 E6 m/s, the initial radial speed being forced to 9.263 E5 m/s. So the initial axial speed is equal to 1.696 E6 m/s. In §4.1 the helix is 0.18 m long with 3.78 turns and a 2 cm radius. The axial magnetic field is equal to 5 T in §4.1 and 4.2 and 2.5 T in §4.3.
The author makes the hypothesis that the axial speed and, consequently the pitch (p), evolves linearly with the distance (d) along the axis (see §3.2), from its initial value (i.e. p = p initial + K .d, with K=dp/dy). The initial pitch (p initial) at the helix input takes into account the initial axial speed (1.696 E6 m/s). The pitch at the helix output takes into account the ion speed Vi (1.932 E6 m/s), because it is considered that all the speed is now axial (i.e. complete transformation of the radial speed into axial speed).

The linearity of the pitch p directly leads to the determination of:

- The number of turns n =L/p_mean, with L the helix length and p_mean the helix mean pitch.
- The wire length Lw = (L^2 + (2.π.n.a)^2)$^{1/2}$

4.1 First test

A first test has been done by forcing the phase of the radial field on the ion phase, which simulates the ideal case (which cannot occur in real life). It can be observed that the rotational movement is gradually slowed down, as expected, with small oscillations of the radial speed due to the fact that the ion trajectory is not centered on the axis.

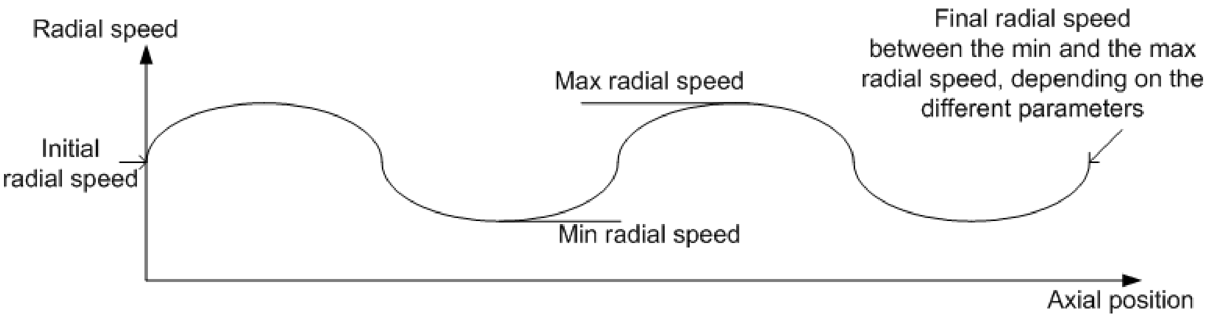
Below, according to the current through the helix, it is given the final radial speed (Vr) and the final axial speed Va. It can be seen that the minimum of Vr is obtained at 44500 A, the final radial speed having been reduced by a factor 10, approximately.

```
10000 A...Vr final= 6.927E+0005  Va final= 1.804E+0006

20000 A...Vr final= 5.133E+0005  Va final= 1.863E+0006

30000 A...Vr final= 3.454E+0005  Va final= 1.901E+0006

40000 A...Vr final= 1.569E+0005  Va final= 1.926E+0006

44500 A...Vr final= 8.966E+0004  Va final= 1.930E+0006

50000 A...Vr final= 2.587E+0005  Va final= 1.915E+0006
```

4.2 Tests with short helixes

Other tests have been done without any constraint on the difference of phase between radial field and ion trajectory. So the ion trajectory is complex.

What it appears clearly is that the radial speed evolves typically as shown below.



In the figure above, the radial speed first increases before decreasing, but it could be the opposite, according to the initial difference of phase between the radial field and the ion (see §1).

The gradient of radial speed evolution depends on the radial magnetic field applied and consequently on the current across the corkscrew. Note that the ion Larmor radius depends on the axial magnetic field.

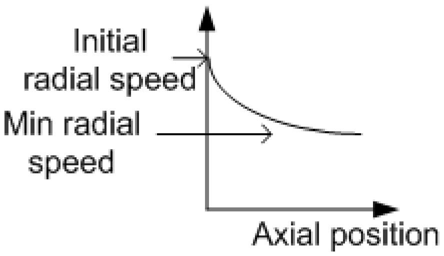
For example, for a given configuration at 5 T and an initial radial speed equal to 9.26E5 m/s for a total speed equal to 1.932 E6 m/s and a corkscrew of radius 0.02 m and length equal to <u>0.16 m</u> (3.36 turns):

- 1000 A will make decrease the radial speed from 9.26E5 m/s to 8.89E5,
- 10000 A will make decrease the radial speed from 9.26E5 m/s to 5.45E5,
- 50000 A will make decrease the radial speed from 9.26E5 m/s to 4.28E5, then make increase it up to 1.25E6 m/s.

So for any particular configuration, after many tests, it will be found a solution of “corkscrew” to reduce the radial speed (but not to eliminate it), the configuration being defined by:

- one speed of ion (monoenergetic),
- a constant ion speed direction (i.e. a constant initial radial speed),
- a constant ion position at the helix entrance.

A reduction of half the initial radial speed is a reasonable target. As shown on the previous figure, it is useless to dispose of many turns. Only a few turns (let’s say inferior to 4 and much probably inferior to 2) are sufficient to reach the first minimum, as shown below.



Note that for these very short helixes, it can be used constant pitch, the results with this type not being very inferior to the variable pitch helixes.

The problem given and solved in figure 8 of reference [1] is typical. The trajectory of the ion is predictable and its injection speed is supposed fixed. So it exists a solution based on a “corkscrew”.

Now, if this solution works for this configuration, it will not work for another configuration (different ion speed, direction or position). The result can be, possibly, the opposite of the one expected (i.e. more radial speed instead of less).

5) About the electrical power consumed by the “corkscrew” and the dissipation of this power

A maximum current density through the coil must be considered to avoid an excessive temperature (>200 °C) in the wire. After some tests, a maximum of about 7 A/mm^2 might be respected for this type coil.

Moreover, the user must assess more precisely the balance between electrical power generated and power dissipated, as described below, to determine the real electrical power consumed and the expected mean temperature of the coil.

It is reminded that the electrical power Pe consumed is equal to $Pe = R.I^2$ where $R = \rho . Lw / Sw$ with:

- ρ the ohmic resistivity of copper. ρ =17.2E-9 Ω.m at 20 °C, which slowly increases with temperature: ρ = 17.2E-9 . (1+ 3.8E-3 . (T – 293)), with T in °K. For example, ρ = 20E-9 Ω.m at 60°C.
- Lw the wire length (m),

- Sw the wire section (m^2) = $\pi \cdot rw^2$, with rw the wire radius.

To cool the coil in the vacuum, there is no convection, but only conduction through the coil stand (on both ends) and radiation. The following description is rough but sufficient for an assessment.

About the power dissipated by conduction (Pc) through the coil stand, Pc is roughly equal to:

$Pc = \lambda \cdot (Tc-Ts) \cdot Sw / Lc$

- λ the thermal conductivity equal to 384 W/(m.K) at 18 °C for copper,
- Tc the mean coil temperature in °K, Ts the stand temperature in °K (supposed equal to 293 °K)
- Lc the mean distance (in m) between the coil and the stand. Let's suppose a mean value Lc = Lw/4

About dissipation by radiation, according to the Stephan-Boltzmann law the radiated power (Pr) is equal to

$Pr = \sigma \cdot \epsilon \cdot (Tc^4 - T0^4) \cdot Sl$, with:

- σ the Stefan-Boltzmann constant equal to 5.67E-8 W/($m^2.K^{-4}$)
- ϵ the emissivity between 0.04 for polished copper and 0.87 for oxidized copper, with 1 for the perfect black body. Let's suppose an emissivity of 0.8.
- Tc the mean coil temperature in °K, T0 the room temperature in °K (293 °K in general)
- Sl the lateral coil surface radiating in $m^2 = 2 \cdot \pi \cdot rw \cdot Lw$, for a round wire.

In permanent behavior, the electrical power must be dissipated by conduction and radiation (Pe=Pc+Pr).

The current being very high to produce the necessary radial field, to reduce the power consumed, the wire radius rw must be relatively important, which can be a problem in front of the helix radius, by hiding the way of particles. A rectangular section for the wire could be a solution. In that case, it would be interesting in the radial field calculation, to take into account the reduction factor K introduced in the formula page 4 of the reference [1], i.e.: $K=\sin(\pi.w/p) / (\pi.w/p)$, with w/p the fraction of cylinder surface covered by the ribbon.

For example, for the corkscrew of 0.16 m length in §4 .2, with a helix radius "a" of 20 mm, a mean pitch "p" equal to 47,6 mm, a wire length "Lw" equal to 0.451 m, a current of 1000 A and a limit of 7 A/mm^2, the wire section must be equal to 143 mm^2. This means a wire radius "rw" of 6.7 mm which is relatively high in front of the helix radius (20 mm). So a rectangular section would be better here.
The power consumed (Pe) will be equal to 80 W and the coil temperature will be equal to 144°C.

Note that a current of 10000 A will give a wire section of 1429 mm2, so the wire radius will be equal 21.3 mm, i.e. superior to the coil radius, so such configuration is not physically possible at a standard temperature. Of course, copper cryogenically frozen or the use of a superconducting material at low temperature will remove this problem.

6) Conclusion

To focalize a particles beam with always the same characteristics (speed, direction and position at the helix entrance), the magnetic corkscrew is a solution to reduce the radial speed, but not to remove it completely. However, the necessary very elevated current at standard temperature limits drastically the interest of such system (see §5).

To focalize a beam with particles having different characteristics (speed and position), the magnetic corkscrew is not the solution and can strongly reduce or increase the radial speed according to each particle.

Reversely, to transform part of the axial speed in radial speed, as shown for example in the figure 4 of reference [1], this device is a solution.

7) References

[1] US patent, number 3,197,680, patented July 27, 1965 by Richard C. Wingerson
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[2] "Fundamental limitations on plasma fusion systems not in thermodynamic equilibrium » by Todd Harrison Rider
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[4] “Electron motion in a helically perturbed magnetic field” by Ajit Singh and Ward D. Getty in IEEE Transactions on Electron Devices 21(1):93 - 100 · February 1974
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