Electrostatic lens sizing

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Revision C

CONTENTS

<u>Revision C</u>: correction of bugs and addition of some details

1. <u>**Goal, presentation and notations used**</u>

The goal of this presentation is to give some additional information about the sizing of electrostatic lenses (i.e. focal length), used to focus particles beams.

The main need of the author was to rapidly size simple lenses, but the different documents found from Internet did not enter in details and did not permit to size the lenses according to the different parameters (voltages and configuration) in a relative simple way, without simulations.

This presentation relies on the Multiplasma simulator program version 1.12 (not public) developed by the author and used for the simulation of the lenses (among other things).

So this paper will permit to size an electrostatic lens by giving an order of magnitude of the focal length.

Notations

- the simple product is indicated with « * » or « x » or « . » or is not indicated if there is no ambiguity,
- the powers of ten are indicated with Ex or 10^x (for example 10^{-7} or E-7),
- the other powers are noted ^ (for example x^2 for x^2) or "SQR(x)" for x^2 or SQRT(x) for x^0.5 (square root).
- "<<" for "very inferior" and ">>" for "very superior"
- |x|, absolute value of x

The author uses SI units or sub-multiple: the unit of length taken here is the "mm" which is more convenient that the "m" for lenses.

Notes:

- the resulting focal length depends on SQR(Ua/Ul), not on the absolute value of Ua or Ul (see §2 for meaning of Ua and Ul),
- breakdown problem between electrodes is not addressed.

Hypothesis:

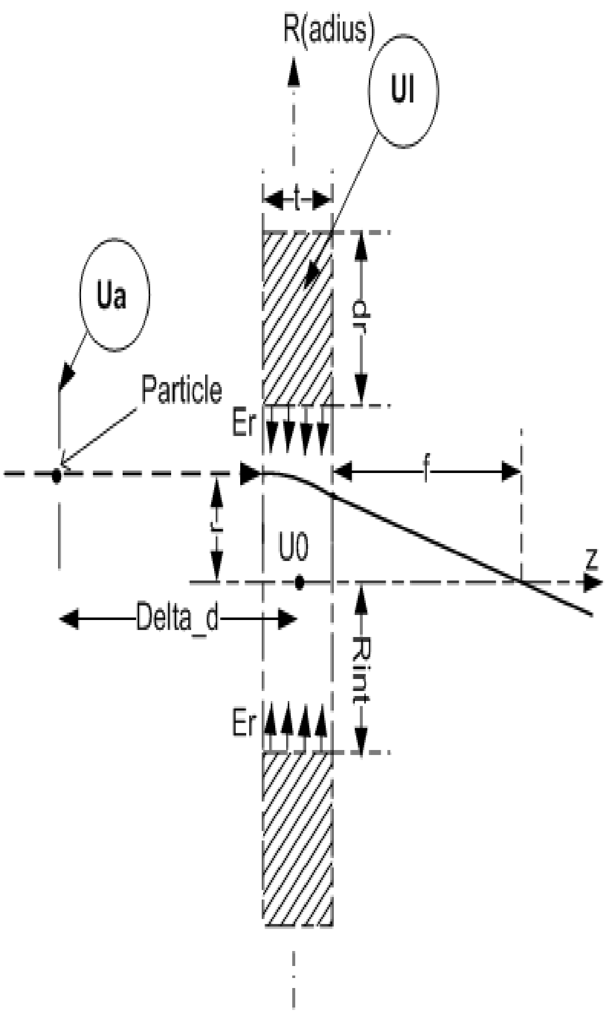
- It is supposed that the initial particle trajectory is horizontal. However due to the not nil beam emittance, the beam is normally more or less divergent, so with a non nil radial speed. This will not affect the calculated focal length.

- It is supposed that the space charge can be neglected. Note that if the space charge is large, its effect is predominant and the lens is without any use.
  The maximum "divergent" space charge radial electric field Ec (V/m) can be determined for a current I (A) of particles beam of radius R (m) and speed V (m/s), by the formula:
  Ec =I/(2.Pi.ϵ0.V.R) with ϵ0=8.85E-12 F/m
  This value of Ec must be very inferior to the "convergent" radial electric field Er generated by the lens. A very rough estimation would be based on a U0 equal to Ul/2. So the rough mean radial Er along the radius would be equal to Ul/(2*Rint).
  If Ec<<Er the lens can be of some use.

2. Description of the simple lens

The goal of the lens is to focus a beam. Due to the interest of the author, it will be supposed that the particles are ions with one charge (D+ or T+). But it could be electrons with opposed voltages. Below the diagram shows the different notations. The simple lens is in fact a single electrode, in form of washer. This type of lens can also be called “Wehnelt”. Physically, the positive voltage of the lens pushes the ions towards the center (through the radial electric field Er).

Simple lens diagram



Ul: voltage (V) applied on the lens
Ua: acceleration voltage (V) applied to the particle. It is equivalent to the energy in eV at which the particle (supposed with a single charge) is sent.
t: thickness (mm)
dr: exterior radius – interior radius (mm)
Rint: interior radius (mm)
f: focal length, in absence of space charge (to calculate) (mm)
r: radius at which the particle penetrates the lens (mm)
Delta_d: distance between the injected particles and the lens center (mm)
Er: radial electric field (V/mm)
U0: induced potential at the lens center (to calculate)

Notes:
* if the particle is an electron, Ua is positive and Ul is negative, with |Ul|<<|Ua|
* if the particle is an ion, Ua is negative and Ul is positive, with |Ul|<<|Ua|

3. Simple lens sizing

We are going, first, to determine a simplified theoretical formula for the focal length, with the first following hypothesis:

- The lens has a cylindrical symmetry, so the azimuthal behavior is not taken into account.
- The trajectory of the particle is initially horizontal (along and above the z axis),
- The distance "r" to the axis is very small compared to the lens radius (Gaussian hypothesis). r is positive by agreement, the axis R carrying "r" being upwards. With this Gaussian hypothesis, the focal length will not depend on "r". In the reality it is worth, roughly, on the first half of the lens radius. Beyond, the beam converges with different smaller focus lengths (causing aberrations).

It can be demonstrated that the trajectory of the ion obeys to:
$d^2r/d^2z=-q/(m.SQR(vz)) . (\delta V/\delta r - \delta V/\delta z.(dr/dz))$

With r the radius, z the axial distance along the z axis, q the charge in Coulomb, m the mass in kg, vz, the axial speed, V the potential (which depends on r and z).

For details, look at the reference [1], page 15.

Here it will be done the hypothesis that dr/dz (i.e. the angle of the trajectory) will be always very small so as to neglect the second term. This supposes that Ul/Ua will be small.

So the expression can be simplified in: $d^2r/ d^2z=-q/(m.SQR(vz)) . (\delta V/\delta r)$.
Note that $\delta V/\delta r=-Er$ (the radial electric field)

In addition, it can be demonstrated (divergence theorem) that:
$\delta V/\delta r=1/2 * (\delta Ez/\delta z).r$

With Ez the electric field along z.

For details, look at the reference [1], page 17.

Moreover, m.SQR(vz)=2* Ek (Ek being the kinetic energy),
If Ek if expressed in eV (and not in Joule), it follows:
$m.SQR(vz)/q=2*Ek_{eV}$ (expressed in eV) or $q/(m.SQR(vz))=1/(2*Ek_{eV})$

In what follows Ek_{eV} will be replaced by Ua (in V).

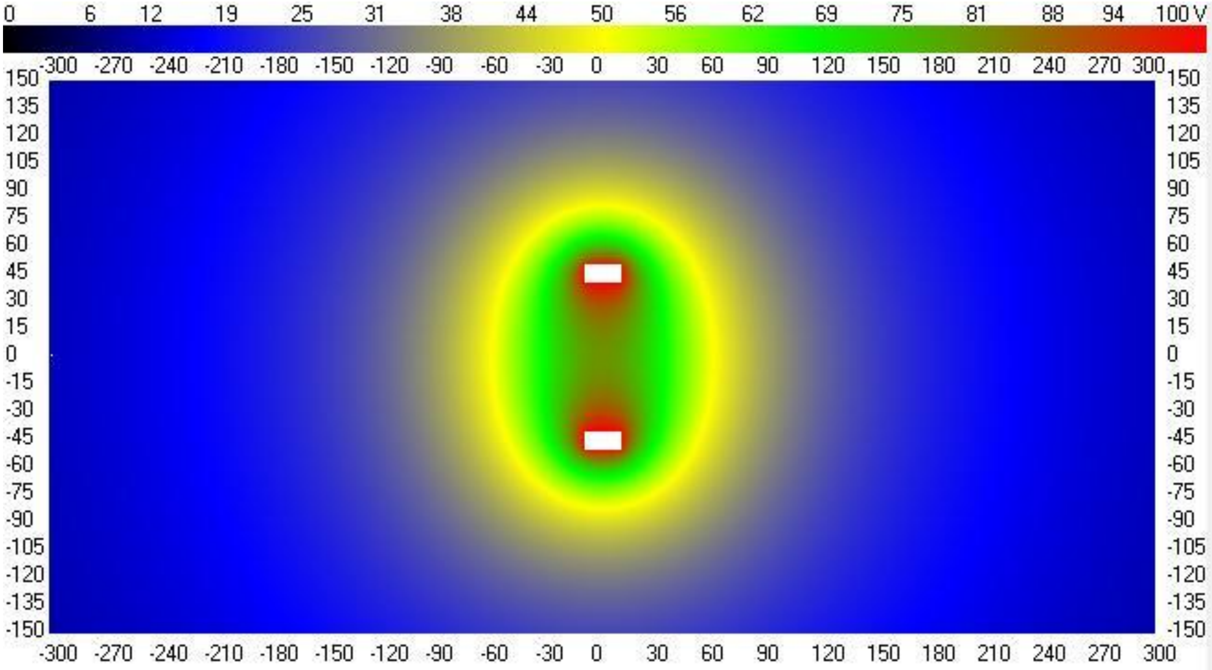
<u>Note</u>: normally Ek (in J)=q . Ua (V) but with a one charge ion, Ek(in eV) = Ua (numerically), Ua being the acceleration voltage of the particle.

So will all these considerations:
$d^2r/d^2z=-1/2*1/2 * (\delta Ez/\delta z).r/ Ua = -1/4* (\delta Ez/\delta z).r/Ua$

By integration along z, we have dr/ dz=-Ez*r/(4*Ua), equal to the angle α if dr/dz is weak (tang(α)≈α). With α , it can be determined f (focal length)=|r/ α|

Moreover, Ez (called Ez1) is negative before the lens, and Ez (called Ez2) is positive after the lens, with normally Ez2=-Ez1
←----Ez1---- Lens at + U0 (V) ----Ez2-----→

Look at the electric field of the lens below (in term of induced potential in V). It appears clearly that, circulating from left to right along z, Ez changes of sign at the center of lens. Note that the electric field tends to become as a spherical electric field when the radius increases.



It can be observed that, from left to right on the axis, the induced potential increases up to U0 at the center of the lens and then decreases from U0.

From all these elements, it can be shown (reference [2] page 116) that
f=4*SQR(Ua)/(Ez*Ul)
<u>Note:</u> as Ez is proportional to Ul, it follows that f is proportional to SQR(Ua/Ul).

Ez is not known but it can be determined experimentally. Moreover for a given lens, U0=K.Ul (K determined experimentally according to t, Rint and dr).

About Ez, experimentally, at D1=2*(Rint+2/3*dr)+1/2*e, the induced voltage is relatively constant around 0.416 U0 for the considered lenses. This gives a reference Ez0=0.584*U0/D1.
The real Ez undergone depends mostly on the distance Delta_d. It is equal to Ez=K2*Ez0, It remains to determine the constant K2 experimentally (according to Delta_d, Rint and dr). Note that it is implicitly supposed that the electric field after the lens center is symmetrical of this one (which is wrong but necessary to remain simple).

Finally, f (mm) = 4*SQR(Ua)*D1*SQR(K)/(K2*0.584*U0/K*U0/K), or
f (mm) = 4*SQR(Ua/U0)*D1*SQR(K)/(K2*0.584)
In the program the constant 4*SQR(K)/(K2*0.584) is replaced by the constant 3.36*K1, so f (mm) = 3.36*K1*SQR(Ua/U0)*D1, but it comes to the same, as the K1 is determined to match, at best, the simulations (supposed to exactly correspond to the reality) with the calculated f.

The complete calculation with the experimental formulas for U0 and K1 is given in the Pascal (Delphi 6) procedure in Appendix.

So given Ul, Ua, Rint, dr, t and Delta_d, it will be given the probable focal length (f) with an estimated dispersion (compared to simulations) from 1/2 to 2 times the result.

An example is given in the program in Appendix.

The limits of validation are the following:

- Delta_d between 1 and 10 times the lens exterior diameter. Note that the influence of the acceleration electrode at the voltage Ua is not taken into account (neglected),
- dr inferior or equal to Rint
- e inferior or equal to 2*Rint
- Ul<=0.4*Ua

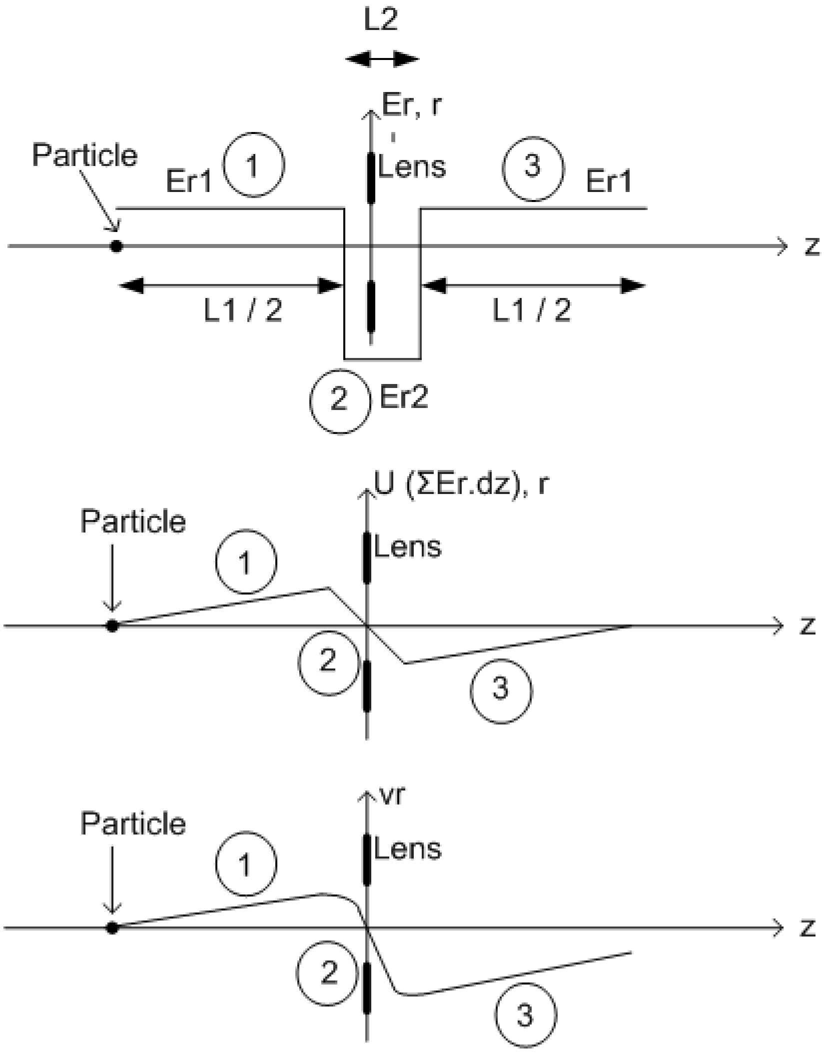
Physical explanation of the proportionality of f with SQR(Ua/Ul)

Note 1: in this chapter, “K” means “any positive value of any dimension” (this to avoid useless developments). Moreover, “K” is not submitted to algebraic operations.
Note 2: ions are only considered in this chapter, this to always have K positive,
Note 3: the sign “~” is worth for “proportional”

Intuitively, it seemed natural, for the author, to think that the focal length f be proportional to Ua/Ul for the following (bad) reason:

> Er (the radial electric field, convergent so negative) is proportional to Ul-U0 and finally to Ul (U0 being proportional to Ul).
> So the radial speed vr=K.Er.cd with “cd” for “crossing duration”.
> cd=t/vz (t: thickness and vz: axial speed along z). Hence vr ~ Ul/vz.
> The angle α=vr/vz ~ Ul/SQR(vz) or Ul/Ua (because vz~SQRT(Ua))
> And finally f=|r/ α|~ Ua/Ul
> But it is not true, because f is indeed proportional to 1/SQR(Ua/Ul).

Looking the evolution of the electric field along the trajectory, it can be noted that for the most part of the trajectory the electric field is low and positive (divergent), whereas at the lens level and a bit around it, the electric field is high and negative (convergent).
In the diagram of the next page, it is shown the rough and simplified shapes of the electric field Er, its integration along the trajectory ΣEr.dz and the radial speed vr.



It can be noted that ΣEr1.dz=- ΣEr2.dz so Er1*L1=-Er2*L2 or Er2=-Er1.L1/L2 (with L1>>L2).

The parts 1 and 3 (Er1>0) are crossed by the particle at a mean speed vz1 along z: vz1=K.SQRT(Ua-U0/2), so the time t1 to cross the parts 1 and 3 is equal to t1=L1/vz1.
The total positive speed vr1 (upwards) got by the particle when it crosses the parts 1 and 3 is equal to vr1=K.Er1.t1 (based on the classical formulas:
v= acceleration.t and Force=q.Er=m.acceleration).
So vr1=K.Er1.L1/vz1

The part 2 (Er<0) is crossed by the particle at a mean speed vz2 along z: vz2=K.SQRT(Ua-U0), so the time t2 to cross the part 2 equal to t2=L2/vz2.
The total negative speed vr2 (downwards) got by the particle when it crosses the part 2 is equal to vr2=K.Er2.t2 or vr2=K.Er2.L2/Vz2=-K.(Er1.L1/L2).L2/vz2=-K.Er1.L1/vz2

The final speed vr=vr1+vr2=K.Er1.L1.(1/vz1-1/vz2)=K.Er1.L1(vz2-vz1)/(vz2.vz1)
Noting that U0<<Ua and so using SQRT(1-Epsilon)=1-Epsilon/2, it comes, neglecting U0 in front of Ua: vr=-K.Er1.L1.U0/(Ua^1.5) which is negative and so convergent (towards the axis).

The angle α of the particle trajectory is equal to $\alpha = vr/vz\_final$
with $vz\_final=K.SQRT(Ua)$.
So $\alpha = -K.Er1.L1.U0/(Ua^{1.5})/(K.SQRT(Ua))=-K.Er1.L1.U0/SQR(Ua)$
As $Er1=K.(Ul-U0)$, with $U0=K.Ul$, it means that $Er1=K.Ul$
Finally $\alpha = -K.SQR(U0/Ua)$ or $\alpha = -K.SQR(Ul/Ua)$

<u>So f (focal length)=|r/ α| is proportional to SQR(Ua/Ul) or SQR(Ua/U0), as expected.</u>

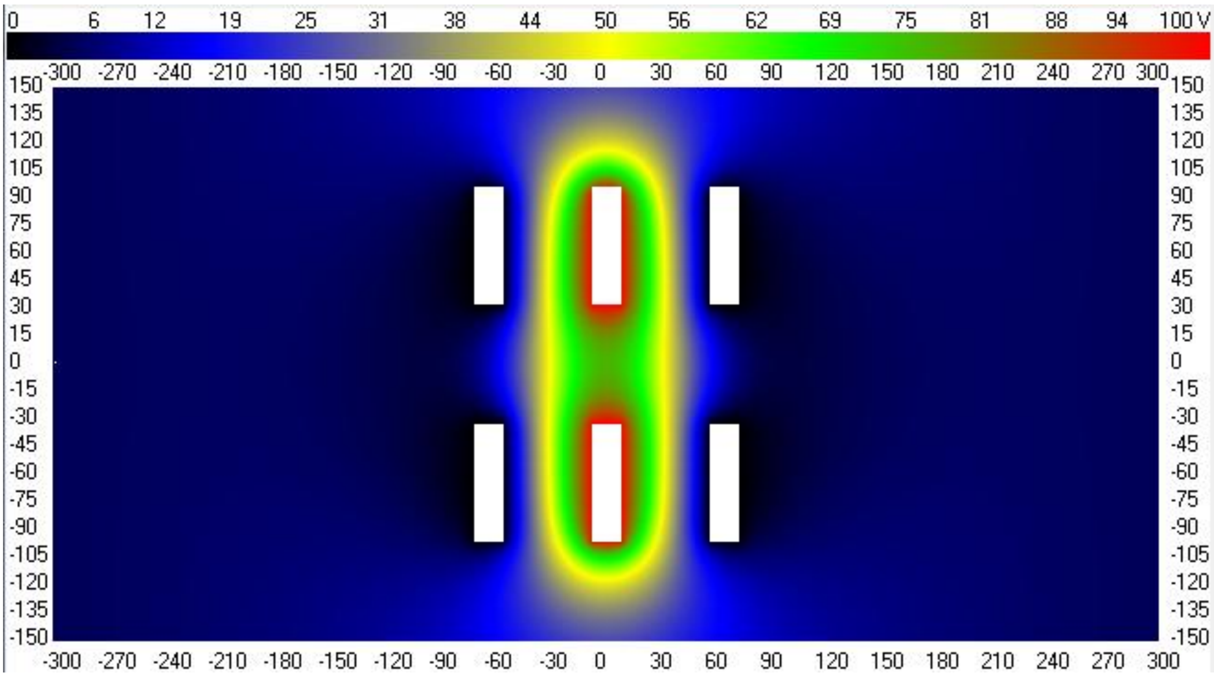
4. About lenses with 0V symmetrical electrodes

It can be simpler to force the electric field with two electrodes at 0 V, symmetrically disposed. The number of parameters being high, it will not be proposed a sizing, but two examples (type 1 and type 2).

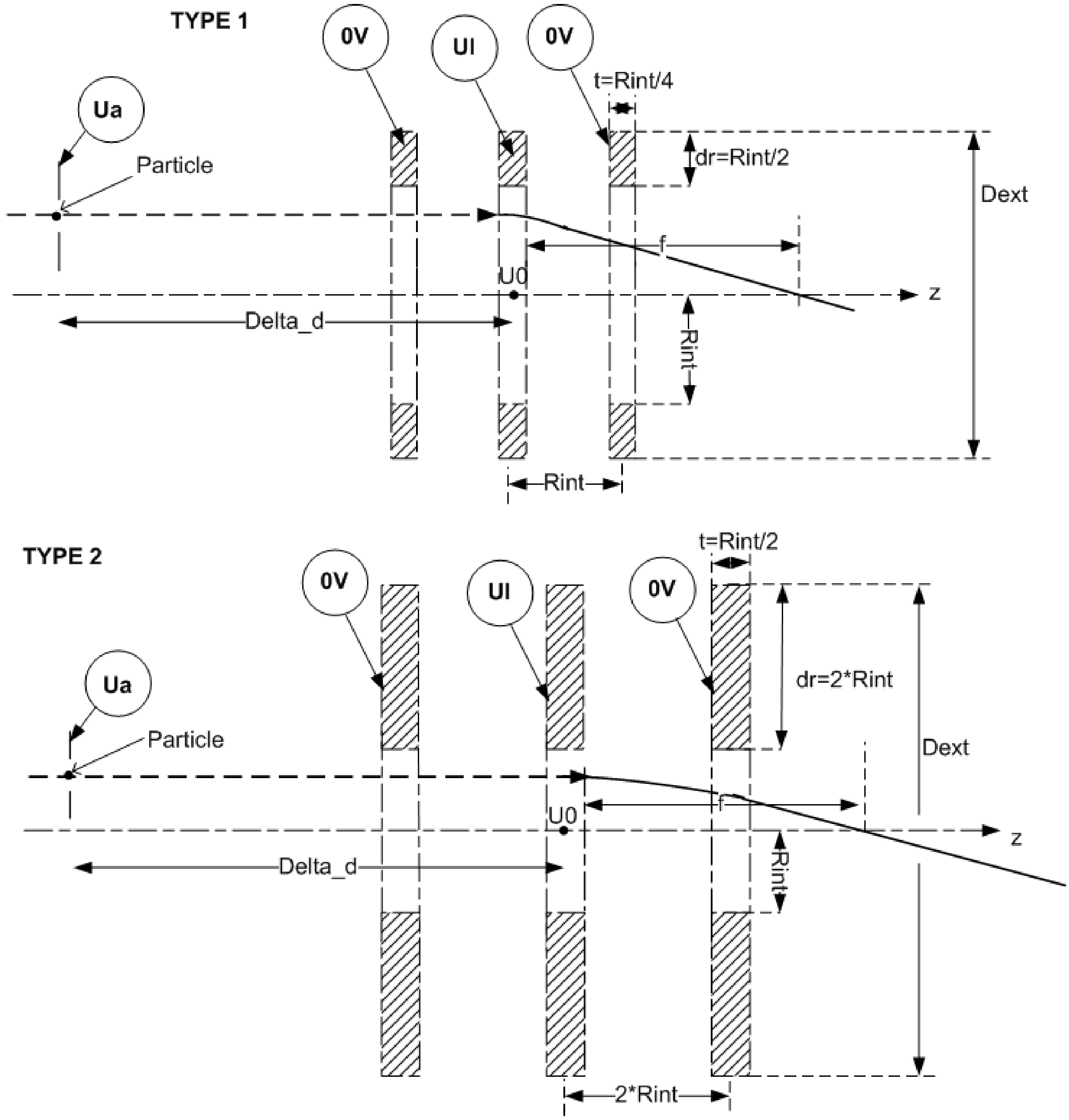
The big advantage of that type of lens is that it does not depend much on the distance between the injected particles position and the lens center (Delta_d), contrary to the single lens. It is almost an isolated system

In the following page, it will be found the diagram of these two examples of lens.

Bellow is an example of electric field for such lenses (in term of induced potential in V). It is obvious, compared to the previous one for a single electrode that the electric field is well limited by the two symmetrical electrodes at 0 V.



Two examples of lenses with symmetrical 0 V electrodes



Ul: voltage (V) applied on the lens
Ua: acceleration voltage (V) applied to the particle. It is equivalent to the energy in eV at which the particle (supposed with a single charge) is sent.
t: thickness (mm)
dr: exterior radius – interior radius (mm)
Rint: interior radius (mm)
Dext: lens external diameter (mm)
f: focal length, in absence of space charge (to calculate) (mm)
Delta_d: distance between the injected particles and the lens center (mm)
U0: induced potential at the lens center (V)

Notes:
* if the particle is an electron, Ua is positive and Ul is negative, with |Ul|<<|Ua|
* if the particle is an ion, Ua is negative and Ul is positive, with |Ul|<<|Ua|

The focal length f1 for the type 1 is given experimentally by the formula:

f1 (mm) = 147 * (Rint/8) * ((Delta_d/(3*Dext)) ^ 0.1) * SQR(Ua/Ul)

As it can be seen, due to the small 0.1 power factor, the influence of Delta_d is weak (but still monotone).

The focal length f2 for the type 2 is given experimentally by the formula:

f2 (mm) =40 * (Rint/4) * SQR(Ua/Ul)

As it can be seen, the influence of Delta_d is not taken into account because it is weak (< to 15%) and, overall, not monotone.

5. <u>Conclusion</u>

- The focal length estimate (f) of the single lens (diagram in page 3) is proposed in Appendix.

- The focal lengths of the type 1 and type 2 lenses with 0 V symmetrical electrodes (diagram in page 9) are proposed just above.

The dispersion of the result (compared to a simulation which has also a certain margin of error) is estimated to be between 1/2 and 2 times the result.

6. <u>References</u>

[1] « Etude théorique et expérimentale de la focalisation des ions afin d’améliorer la brillance du faisceau ionique par suppression des causes d’aberrations » by Jean Faure

[2] « Sur une nouvelle méthode de focalisation des faisceaux d’ions rapides. Application à la spectrographie de masse » by Louis Cartan

APPENDIX

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PROCEDURE SIMPLE_LENS_SIZING_FOR_IONS;
VAR t:SINGLE;{"t" for "thickness of the lens" in mm}
VAR Ul:SINGLE;{voltage (in V) applied on the lens (positive for ions and negative for electrons)}
VAR Ua:SINGLE;{accelerator voltage (in V) applied on the particles (negative for ions and positive for electrons)}
VAR Rint:SINGLE;{interior radius in mm}
VAR Dext:SINGLE;{exterior diameter in mm}
VAR dR:SINGLE;{exterior radius - interior radius in mm}
VAR U0:SINGLE; {induced potential at the lens center (to calculate)}
VAR Delta_U:SINGLE;{|U1-U0| in V}
VAR D1:SINGLE;{standard distance compared to the lens center in mm}
VAR Delta_d:SINGLE;{distance between the injected particles and the lens center in mm}
VAR K1:SINGLE;{coefficient to take into account Delta_d, for the Ez estimation}
VAR f:SINGLE;{focal length to calculate in mm}
BEGIN
 {set of hypothesis, for ions}
 Delta_d:=99;
 Ul:=40000;
 Ua:=-100000;
 dR:=2;
 Rint:=9;
 t:=4;

 {not permitted}
 IF (Ul<0) OR (Ua>0) THEN EXIT;
 IF ABS(Ul)>ABS(Ua) THEN EXIT;

 {calculation}
 Delta_U:=Ul*0.2479*Power(2.25*t/Rint,-0.2863-(t/Rint)*0.4657)*Power(4.5*dR/Rint,-0.2852-(dR/Rint)*0.0891);
 U0:=Ul-Delta_U;
 Dext:=2*(Rint+dR);
 D1:=2*(Rint+2/3*dr)+t/2;
 IF Delta_d<=3*Dext THEN K1:=Power(Delta_d/(3*Dext),0.6-(Delta_d-3*Dext)/(6*Dext))
 ELSE K1:=Power(Delta_d/(3*Dext),0.34);
 f:=3.36*K1*D1*SQR(Ua/U0);

 {display of f}
 //For the previous set of hypothesis, the result is f=966 mm;
 WRITELN('f=',f:4:0,' mm');
END;
```