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13th of November 2025 – Revision C

Physics, dimensioning, and experimentation with data transmission via magnetic induction, at low frequencies and in the near field

ABSTRACT: this article mainly analyzes the physical aspect of data transmission via the magnetic field in the near field, i.e. at low frequencies, with the aim of proposing relatively simple applicable formulas. It then proposes the best possible dimensioning of the equipment used, in particular solenoids. Finally, tests are carried out to validate the formulas used and, ultimately, an experiment between an FT8 transmitter and receiver is proposed. This article is relevant to amateur radio operators for possible experiments and, of course, to caving and underground work.

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1. Introduction

For a number of applications (remote battery charging, for example), energy needs to be transmitted from one point to another with the highest possible efficiency. Various techniques are used for this purpose, mainly magnetic induction, electrical induction, and electromagnetic transmission through a thin beam.
In this article, we will determine the best way to ensure the transmission of data, rather than energy, by magnetic induction. The problem is different because the criterion for success here, for a certain range, is the best signal-to-noise ratio and not the best energy efficiency associated with optimum impedance matching.

The case of data transmission using only the electric field has already been analyzed by the author ([1]). We will show that magnetic induction is more interesting in cases where matter interferes with the transmission path, such as walls, trees, etc.

It should be noted that this article also concerns the possibility of transmitting through rock, for caving and underground work, since the problem is the same: transmitting information from an underground cavity to the surface, and vice versa.

We are working at low frequencies (0.05 to 138 KHz), under the AQSR assumption (Approximation of Quasi-Stationary Regimes, see Appendix A), i.e. in the “near field.” A stationary regime (continuous regime) is of no interest because it does not allow data transmission. In the AQSR regime, the evolution time of the modulated sinusoidal signal is much longer than the propagation time over the transmission path (in other words, the magnetic field instantly follows the evolution of the source). As a result, the coupling between electrical and magnetic phenomena explained by Maxwell's equations is simpler (see Appendix A).

Notations
In the rest of the text:

- The simple product between scalars is denoted by “.” or “x.”
- In formulas and equations, the operators x and / take precedence over + and – . Therefore, A x B + C is understood to mean (A x B)+C.
  Similarly, exponentiation takes precedence over all operators. Therefore, A x B^2 + C is understood as (A x (B^2))+C.
- “§” for ‘paragraph’
- “≈” for "approximately "
- The letter “K” denotes any proportionality term (not to be confused with a constant). This avoids the use of the proportionality sign “~”.
- [x] means “reference number x”, which can be found in the “References” section at the end of the article.
- “xEy” means “x 10^y ” For example, ‘3E8’ and “3 10^8” have the same value.
- “i.e.” means “that is.”
- ε0 and εr are respectively the permittivity of vacuum equal to 8.854E-12 F/m and the relative permittivity of the material.
- μ0: the magnetic permeability of vacuum = 4.π.10^{-7} H/m
- μr: the relative magnetic permeability (dimensionless)
  Note: the product (μ0 x μr) is, for the magnetic field, the equivalent of conductivity for the electric field, i.e. a kind of “magnetic conductivity.”

- c: the speed of light equal to 3E8 m/s.
- w: the angular frequency (in rd/s) equal to 2 x π x f, where f is the frequency in Hz
- λ: the wavelength in m (λ = c / f)
- i: "complex" number such that $i^2=-1$
- |x| is the modulus of the complex number x.
- Vectors are in bold and scalars are in "light" font.

SI units, multiples and sub-multiples are only used

2. Maximum distance between transmitter and receiver justifying the AQSR hypothesis, for a given wavelength

In practical terms, we need to have an order of magnitude for the maximum distance defining the "near field" for induction by a magnetic dipole.

With regard to the magnetic field in the near field, if we consider a magnetic dipole (i.e. a small loop), we find that the orthoradial component Bθ in spherical coordinates is of the type: $B\theta = K \times \left(\frac{1}{r^3} - \frac{i \times w}{r^2 \times c} - \frac{w^2}{r \times c^2} \right)$ ([2] page 387) and that the azimuthal electric field Eφ is of the type $E\varphi = K \times \left(\frac{i \times w}{r^2} + \frac{w^2}{r \times c} \right)$ ([2] page 387), where r is the distance between the dipole ("the transmitter") and the point under consideration ("the receiver"). We can therefore see that if r tends towards 0 (r<<λ), Bθ tends towards $B\theta = K \times \frac{1}{r^3}$ as in stationary regime ([2] page 221) and Eφ tends towards $E\varphi = K \times \left(\frac{i \times w}{r^2} + \frac{w^2}{r \times c} \right)$ while the ratio $\frac{E\varphi}{B\theta}$ tends towards r and therefore towards 0.
The electric field therefore becomes negligible when r tends towards 0, which was to be expected.

> Note: at a large distance (r>>λ), once the electromagnetic field Eφ/Bθ has formed, the magnetic field tends towards $B\theta = K \times \frac{w^2}{r \times c^2}$

To define the near field, we will consider that the term $\frac{1}{r^3}$ of Bθ (characterizing the near field) must be $\sqrt{10}$ times greater than the term $\frac{w^2}{r \times c^2}$ of Bθ (characterizing the far field). Therefore, $\frac{1}{r^3} > \frac{\sqrt{10} \times w^2}{r \times c^2}$ or $r < \frac{\lambda}{\sqrt{\sqrt{10} \times 2 \times \pi}} \approx 0.09 \times \lambda$

For example, if f=3 KHz (λ=100 km), then the distance r must be less than 9 km to remain within the AQSR (near field) hypothesis.

Conclusion
The criterion to be retained is $r < 0.09 \times \lambda$ therefore this one, based on a magnetic field ratio (near field/far field) of $\sqrt{10}$, i.e. an energy density ratio of 10, since magnetic energy density is proportional to the square of the magnetic field (e.g. [2] page 322).

Note that for an energy density ratio of 100, the criterion would change to $r < 0.05 \times \lambda$ and for a ratio of 1, it would change to $r < 0.16 \times \lambda$ (or $r < \frac{\lambda}{2 \times \pi}$)

Note 1: for a small electric dipole (short-distance monopoles), we would have found symmetrical expressions and therefore the same distance criteria.

Note 2: for two electric monopoles at a large distance, the induced voltage varies according to $\frac{1}{r^2}$ between the monopoles, for the main case (see [1]), which does not call into question the near-field distance.

3. Advantage of magnetic field transmission over electric field transmission

It is understood that a vacuum does not dissipate energy as heat. However, the materials through which the field passes can dissipate energy as heat. We will compare the losses generated, depending on the type of transmission.
It should be noted that these losses are not prohibitive in themselves because, in the case of data transmission, the goal is not to transmit the maximum power, but the highest possible induced voltage. But even in this case, these losses must be supplied by the generator. Depending on the power of the generator and therefore its internal resistance, the induced voltage will be affected to a greater or lesser extent: slightly if the generator is powerful and vice versa.

3.1 Loss due to transmission by the electric field through a material, in the near field

The author conducted several tests to quantify the transmission of the electric field through two exterior walls and two interior partitions. The tests were conducted at low frequencies and are assumed to be applicable up to HF frequencies. On the first exterior wall of a new building, the LF signal is completely blocked, with the wall acting as a Faraday cage. This is probably due to the concrete reinforcement bars, which have been grounded.
On the second exterior wall of a building constructed in 1975, the signal is not blocked and the wall acts as a conventional insulator with an εr of approximately 5. The surface capacitance is deduced from the wall thickness and εr and is therefore estimated at 240 pF/m^2. In addition, the 9 cm thick partitions act as insulators with an overall εr between 2 and 3.
If the walls and partitions behaved like perfect insulators, there would be no loss (of heat), but concrete is a poor insulator. In fact, the resistivity ρ of concrete is not infinite but averages around 70 Ω.m ([3]).

To clarify things, let's take a simple example. Suppose that two conductive discs (i.e. the anode and cathode) with a diameter D of 30 mm are separated by a distance L of 300 mm. The separating material consists of concrete between two air gaps.

Furthermore, a real capacitance can be represented by a capacitance in parallel with an insulation resistance, respectively Cac and Rac in the diagram below (see [4]).

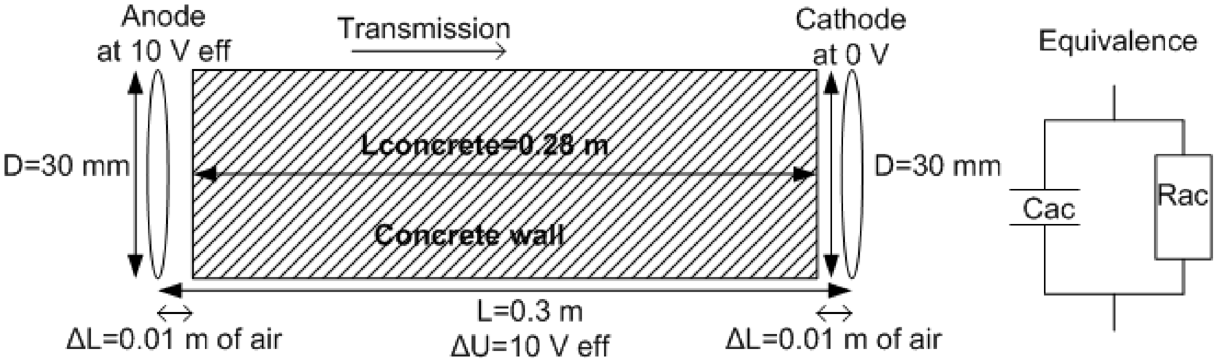

Figure 1: Example of a diagram of a transmission by the electric field

Since (dry) air is a virtually perfect insulator, there is no direct conduction current between the anode and cathode, which does not prevent Joule losses. Let's calculate the electric field Econcrete in concrete, with εr=5. To do this, we simplify the problem by considering it to be a capacitor (i.e. with total influence). We then have:
ΔU = 10 V = Eair x 0.02 + Econcrete x 0.28, with Econcrete=Eair / εr or Eair=Econcrete x εr.
We therefore find Econcrete = ΔU / 0.38 = 26.3 V/m in the example

Since the resistivity ρ of concrete is 70 Ω.m ([3]), the volumetric Joule loss PJv (in W/m^3) in concrete can be estimated at $Pjv = \frac{Econcrete^2}{\rho} = \frac{Econcrete^2}{70}$ =9.9 W/m^3 in the example.
The volume of the concrete wall Vc (between the electrodes) is equal to $Vc = \frac{\pi \times D^2 \times Lconcrete}{4}$ =1.98E-4 m^3 in the example. The Joule loss is equal to PJ = PJv x Vc, or 1.96 mW in the example, which is an equivalent resistance of Rac = ΔU^2 / PJ, or 51.0 kΩ in the example, compared to the resistance of the concrete wall: $Rconcrete = \frac{Lconcrete\rho}{S} = \frac{Lconcrete \times \rho \times 4}{\pi \times D^2}$, or 27.7 kΩ
The air gaps increased the overall resistance, but without completely counteracting the ohmic losses.

3.2 Loss due to transmission by the magnetic field through a material, in the near field, and conclusion

The only losses along the “path” of the magnetic induction flux are those related to eddy currents in the metal parts traversed, where there is then (unnecessary) heat production. These losses are, in principle, negligible.

For the sake of completeness, it should nevertheless be noted that in the formation of the magnetic field, there are ohmic losses in the coils, proximity and skin effects that increase these ohmic losses, and losses due to hysteresis and eddy currents in ferrites, if used.

Conclusion
The voltage induced in transmission by the magnetic field decreases rapidly according to a $1/X^3$ law (X being the distance), whereas it only decreases by $1/X^2$ for transmission by the electric field between an anode and a cathode ([1]).
However, transmission by the electric field through a poor insulator is affected by losses, unlike transmission by the magnetic field. In addition, if ground-connected metal fittings form a Faraday cage against the electric field, transmission is blocked.
Ultimately, transmission via the magnetic field is therefore more advantageous, except in the very specific case where the user wishes to limit transmissions to a space enclosed by a Faraday cage, in which case transmission via the electric field would be more appropriate.

> Note: qualitatively, it appears that it is technically easier to produce a strong magnetic field than a strong electric field. Indeed, the magnetic field requires a strong current and a strong inductance that can extend in a volume whereas an electric field requires a high voltage (more difficult to control than a strong current) and large electrodes that can only extend on the surface. Now it is easier to manufacture a long electric wire than a large metal plate.
> Furthermore, it seems that, in terms of the human body and standards, a strong electric field is considered more dangerous than a strong magnetic field (which simply passes through the body).

4. Theoretical study of transmission through the magnetic field in the near field

4.1 Description of the series configuration

The schematic diagram of this first series configuration (i.e. C1 and L1 in series) is as follows:

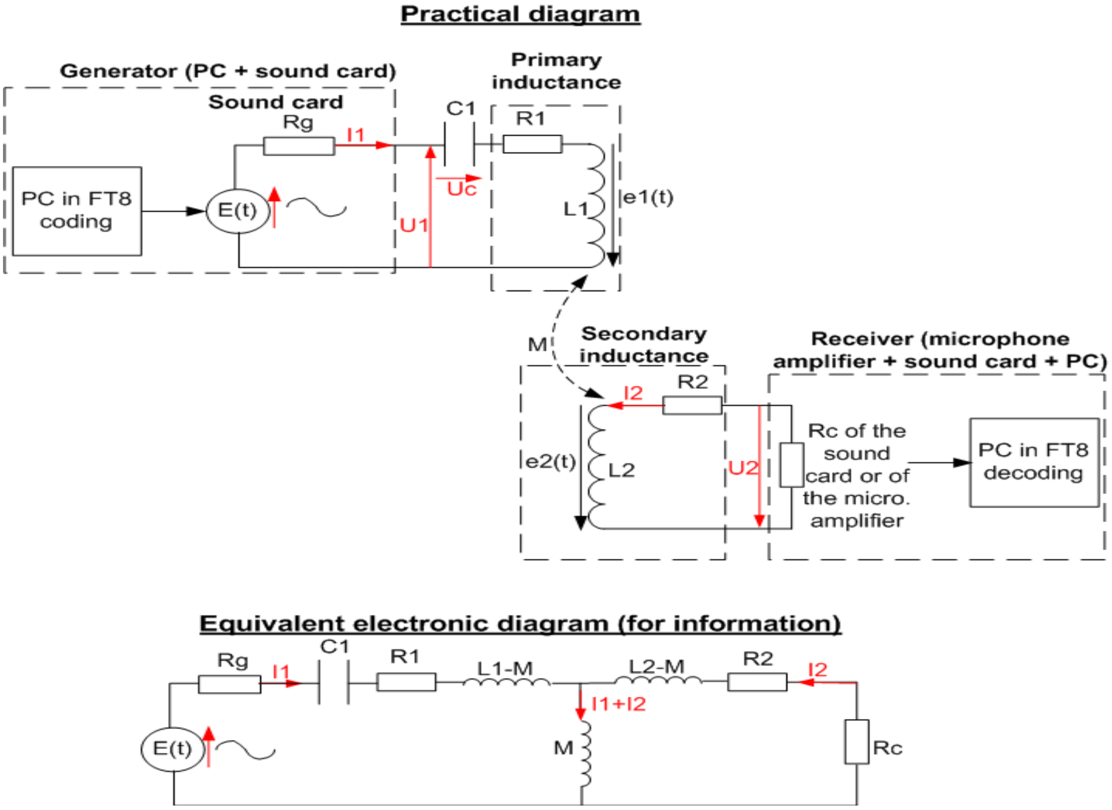

Figure 2: Schematic diagram of the C1 – L1 series configuration

This configuration works as follows: a first PC transmits a message in FT8 via the PC's sound card at an LF frequency (for example, slightly below 3000 Hz).
The signal output from the sound card is sent to the C1/R1/L1 system in series resonance at w=w0 (resonance frequency f0 with w0=2.π.f0). Through magnetic induction (L1/L2 forming a loose coupling), the signal received by the L2/R2 inductor is transmitted to the input of the "microphone" amplifier or sound card (Rc) before being digitized by the sound card. This signal is then decoded in FT8 mode.

> Note 1: most sound cards have a built-in internal "microphone" amplifier (ranging from 0 to 30 or 40 dB). If this is not the case, an external "microphone" amplifier will be required, upstream of the sound card.
>
> Note 2: this is a very simple setup, but sufficient for studying this transmission. It could be improved with an LF amplifier on the generator side and, possibly on the receiver side, with a very low-noise microphone amplifier with high input impedance, equipped with a narrow bandpass filter around the transmission frequency, followed by a high-quality sound card (also low-noise).

The physical description will be based on document [5], which is very comprehensive for those wishing to transmit energy by magnetic induction. We will not discuss the configuration with a non-resonant primary, which, as explained in [5], is not very efficient. The secondary resonance configuration is, in our case, unnecessary because the load resistance Rc is very high compared to the reactance of L2 (L2.w) and R2, and no impedance matching is required. Therefore, only the primary is in series resonance.

This system can be expressed in the form of three equations. The voltage and current values are implicitly expressed in a "complex" form (i.e. in the form z=a+jb=ρ x exp (φ)). They will not be underlined to simplify the notation. The FT8 signal is assumed to be sinusoidal at a frequency w (w= 2 x π x f where f is the frequency), although it is not really sinusoidal since it is modulated, but this is irrelevant here. Therefore, a parameter I or U will be implicitly written as "ρ x exp (j x w x t + θ)" and its derivative will therefore be expressed as the product "j x w". First, a reminder: Φ1 and Φ2 are the "total magnetic fluxes" (to be distinguished from the actual flux φ through a coil φ=B x S) and e1(t) and e2(t) are the induction EMFs. e1(t) and e2(t) are "generating" voltages. Therefore, these EMFs are oriented in the direction of currents I1 and I2.

According to [2] page 280, for two coupled circuits, we generally have (M being the mutual inductance):

$$\Phi 1 = L1 \times I1 + M \times I2$$
$$\Phi 2 = M \times I1 + L2 \times I2$$

By Faraday's law ([2] page 259):

$$e1(t) = -\frac{d(\Phi 1)}{dt} = -(j \times w \times L1 \times I1) - (j \times w \times M \times I2)$$
$$e2(t) = -\frac{d(\Phi 2)}{dt} = -(j \times w \times L2 \times I2) - (j \times w \times M \times I1)$$

Since: $E(t) + e1(t) = (Rg + R1) \times I1 + \left(\frac{I1}{j \times w \times C1}\right)$

Hence $E(t) = (Rg + R1) \times I1 + \left(\frac{1}{j \times w \times C1} + j \times w \times L1\right) \times I1 + (j \times w \times M \times I2)$ (equation 0)
And since $e2(t) = (Rc + R2) \times I2$
Hence $0 = (Rc + R2) \times I2 + (j \times w \times L2 \times I2) + (j \times w \times M \times I1)$
At resonance (w=w0), assuming $Z2 = (Rc + R2) + (j \times w0 \times L2)$, we obtain:
$0 = Z2 \times I2 + (j \times w0 \times M \times I1)$ (equation 1)
Furthermore, $U2(t) = Rc \times I2$ (equation 2)

The goal here is for U2(t) to be as large as possible, in order to obtain the best signal-to-noise ratio at the microphone amplifier or sound card.

> Note: part of the U2 signal consists of ambient noise (interference), but mainly 50 Hz (60 Hz in certain countries) noise. We cannot get rid of ambient noise, but we can place the resonance frequency f0 between two harmonics of 50 Hz (the frequency of the electrical network), so ideally either at f0=xx25 Hz or at f0=xx75 Hz. This will limit the impact of 50 Hz noise on the reception of the FT8 signal, which has a bandwidth of 50 Hz.

With the primary at resonance (at w=w0), we have $\frac{1}{j \times w0 \times C1} + j \times w0 \times L1 = 0$ and, assuming $Z1 = (Rg + R1)$, we obtain from equation 0:
$E(t) = Z1 \times I1 + (j \times w0 \times M \times I2)$ (equation 3)
Starting from equations 1, 2, and 3, after a few calculations we obtain:

$$|U2(t)| = \left|\frac{-j \times w0 \times M \times Rc \times E(t)}{Z1 \times Z2 + (w0 \times M)^2}\right|$$

To draw conclusions from this, we need to simplify this equation.
The input impedance Rc of the sound card (or microphone amplifier), which we assume to be resistive, is between 1.5 kΩ and 50 kΩ. R1 and R2 are in the range of 10 to 20 Ω, L1 and L2 are in the range of a few tens of mH (i.e. approximately 400 Ω of reactance), and Rg of the sound card (headphone output) is in the range of 100 Ω. This same sound card will send the signal with a maximum effective voltage Eeff of 1 or 2 V, as it is only used to power headphones.
M will be less than a few mH (say less than 40 Ω reactance) and w0 will be around 20,000 rd/s (f0=3183 Hz).
Therefore, Z1xZ2 will be much greater than $(w0 \times M)^2$, a term that can therefore be neglected. Since Rc is much greater than R2 and L2 x w, we can write Z2≈Rc
Finally, we can simplify $|U2(t)| \approx \frac{w0 \times M \times |E(t)|}{(R1 + Rg)}$ (equation 4)
To maximize U2(t), E and M must be as large as possible (for M, see §4.4) and R1+Rg must be as small as possible. R1 is the effective resistance of the inductance (see §4.5.4). Rg, the internal resistance of the generator, can be lowered to 4 Ω, or even less if the sound from the sound card is amplified with an LF amplifier. This same amplifier can send the signal with a voltage |E| of at least 10 V. Ultimately, an LF amplifier will introduce an appreciable gain.

Voltage across C1
At resonance (w0), the voltage across C1, |Uc| = |I1| / (C1 x w0), is equal in magnitude to the voltage across L1: |I1| x (L1 x w0).
Furthermore, since M and I2 are very small, the term $(j \times w0 \times M \times I2)$ can be neglected in equation 3. We therefore have

I1≈ $\frac{E(t)}{(R1+Rg)}$ and $|I1| \approx \frac{|E(t)|}{(R1+Rg)}$ (equation 5). Hence $|Uc| \approx \frac{|E(t)| \times L1 \times w0}{(Rg+R1)}$. For example, suppose that the quality factor $Q = \frac{L1 \times w0}{(Rg+R1)}$ is equal to 20, and that |E|=10 V. In this case, capacitor C1 will have to withstand a voltage of |Uc|=10 x 20=200 V, which is high.

4.2 Description of the parallel configuration

The schematic diagram of this parallel configuration (i.e. C1 and L1/R1 in parallel) is as follows:

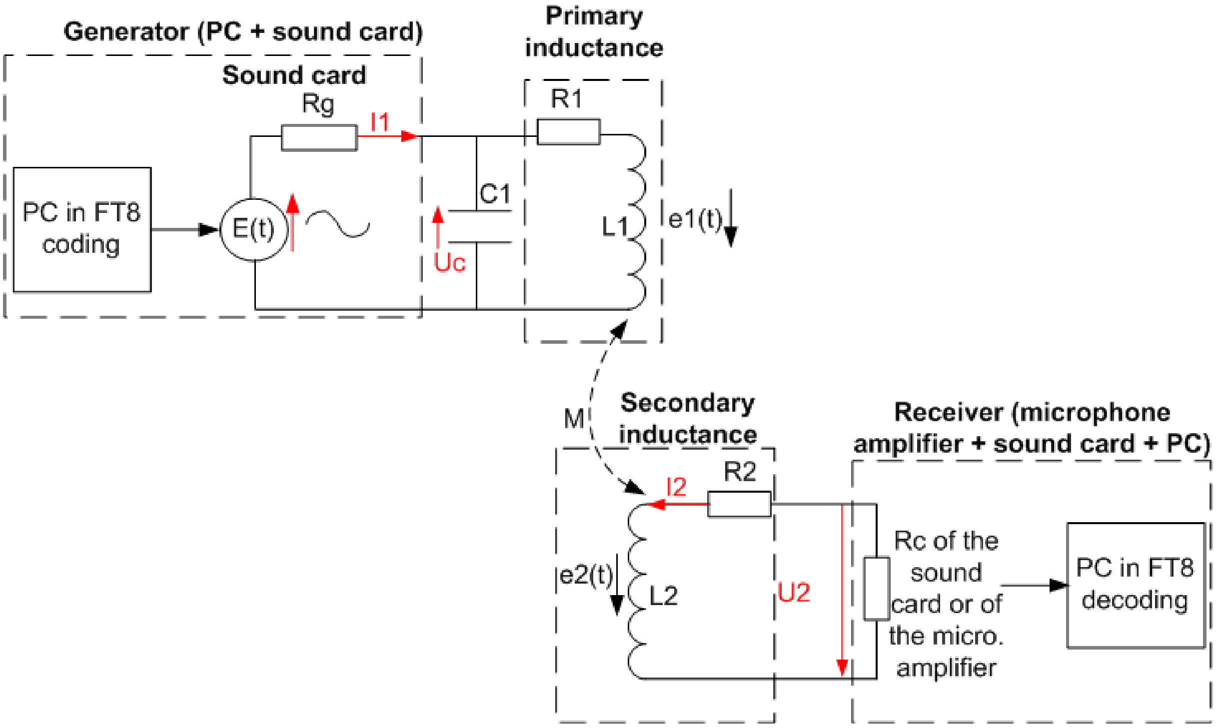

Figure 3: Schematic diagram of the C1 – L1 parallel configuration

Since R1 is much smaller than L1 x w0 at resonance, we can assume that the previous generator circuit (figure 3) and the next generator circuit (figure 4) are equivalent.

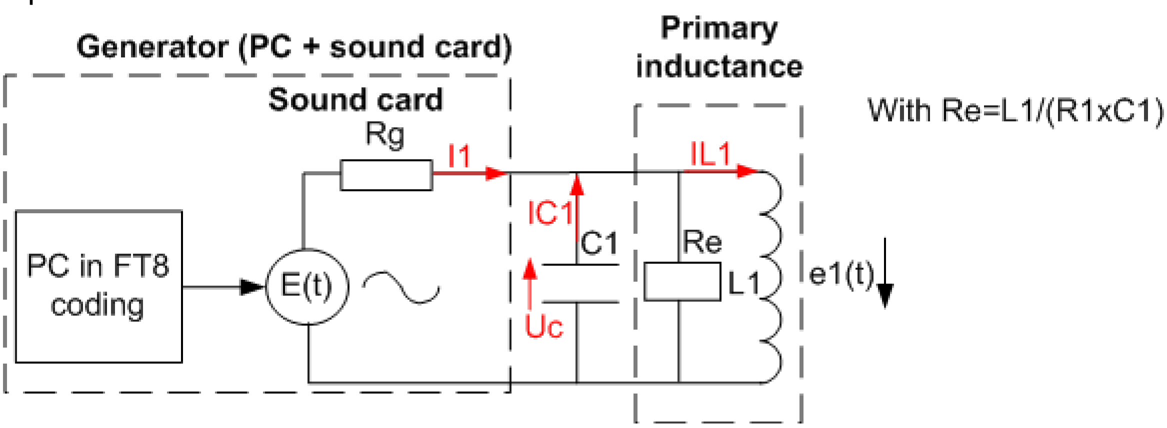

Figure 4: Equivalent diagram of the generator circuit

It can be shown that at resonance (w0): IC1 = -IL1.

The current I1 is therefore $I1 = \frac{E(t)}{Rg+Re}$ $with\ Re = \frac{L1}{R1 \times C1}$

Since Re is much larger than Rg, we can write $Uc(t) \approx E(t)$ and $I1 \approx \frac{E(t) \times R1 \times C1}{L1}$
It can be shown that $|IL1| = Qself \times |I1| \approx w0 \times |E| \times C1 \;\; avec \; Qself = \frac{L1 \times w0}{R1}$

4.3 Comparison between the two configurations

Let us compare this parallel configuration with the series configuration, using the following data, which assumes an LF amplifier: Eeff=10 V, Rg=4 Ω, R1=10 Ω, L1= 10 mH, C1=250 nF, w0=20000 rd/s

In this case:
- I1 in the parallel configuration ≈ 2.5 mA
- IL1 in the parallel configuration ≈ 50 mA
- I1(=IL1) in the series configuration = 714 mA

The current through L1 (and therefore the field created) in the parallel configuration will be lower (50 mA) than that in the series configuration (714 mA), but the capacitor C1 will only be subjected to a voltage of 10 V, whereas in the series configuration, C1 will have to withstand 143 V.
The parallel configuration is not useful. In fact, in the series configuration, if I1=50 mA, then the voltage that C1 will have to withstand will also be 10 V, as in the parallel configuration. However, the series configuration will allow a much higher current to be obtained, so this is the one we will use for the rest of this document.
Note that to form a capacitor of a certain value, capacitors can easily be connected in series to reduce the voltage that each capacitor will have to withstand.

> Note: in [5] §2.d, an intermediate configuration between the series and parallel configurations is proposed. It is not studied here, as it does not seem to be of much interest.

4.4 Determination of the field produced B1 and the mutual inductance M (between L1 and L2)

Reminder: we are in a series configuration (Figure 2).

The parameters B1 and M can be calculated precisely using a finite element method. But here we will settle for estimates.
The magnetic field at any point P located on the axis of a single-layer solenoid is given by the formula $\boldsymbol{B} = \frac{\mu 0 \times \mu r \times N \times I}{2 \times l} \times (\cos\alpha 1 - \cos\alpha 2) \times \boldsymbol{ez}$ ([2] page 202). N is the number of turns and l is the length of the solenoid. We will assume that this formula is applicable to a multilayer solenoid using the average radius to estimate α1 and α2.

After some calculations and assuming that the distance X between point P and the center of the solenoid is much greater than r1 and l1 (see Figure 5), we find the following approximate formula for the field B1 produced by L1 at point P, along the axis of the solenoid: $B1 \approx \frac{\mu 0 \times \mu r1 \times N1 \times I1 \times r1^2}{2 \times X^3}$, where μr1 is the relative permeability of solenoid L1. Note that l1 (the length of L1) does not appear in the formula.

> Note: the B1 field is maximum on the axis and decreases away from the axis.

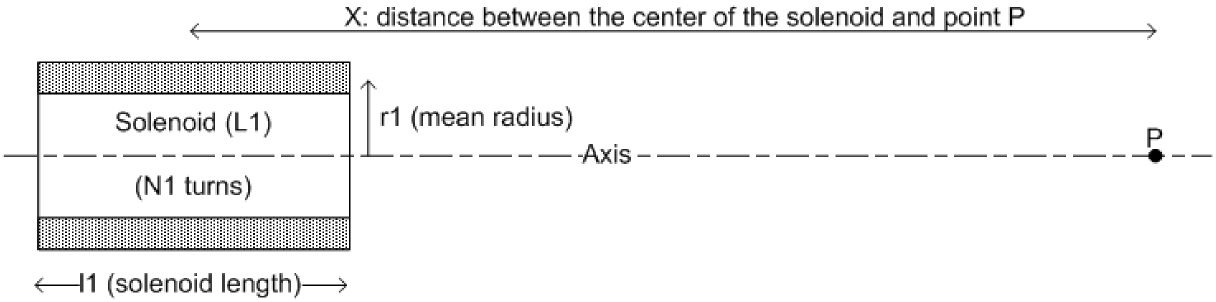

Figure 5: Field at high distance from a solenoid

To estimate the mutual inductance, we will first assume that the inductance L2 at point P consists of a single turn with surface area S2.

Normally, the "total flux from L1 to L2" Φ12 is written as $\Phi 12 = \iint_{S2} B1 \times dS2$

Since loop L2 is distant from L1, at a distance X, we can assume that field B1 is uniform at the level of the turn of L2 and perpendicular to the plane of this turn. In other words, the L2 turn is assumed to be located on the axis of L1 and parallel to the turns of L1. We can therefore write $\Phi 12(X) \approx B1 \times S2$, with B1 estimated at distance X and S2 the cross-section of L2 with S2= π x $r2^2$, r2 being the average radius of L2

Now, for N2 turns and a relative permeability μr2 (for any ferrite inserted in L2), we can write: Φ12≈B1×μr2×N2×S2 (with μr2=1 if there is no ferrite).

> Note: we can see that the external field B1 is multiplied by μr2 because it is concentrated by the action of the ferrite in L2 (if it exists). This is because ferrite has low reluctance (equivalent to resistance for an electric field) compared to the reluctance of air. It therefore acts as an element of low "resistance" for magnetic field lines. At the limit, for μr2→∞, this would be a "magnetic short circuit" and all the field lines from L1 would be concentrated in the ferrite, the leakage field would then be zero. This principle is also used with mu-metal for magnetic shielding in LF.
> It is to be expected that this concentration of field lines will become less and less effective as μr2 increases and therefore that Φ12 will become less and less linear with μr2, because the leakage field tends asymptotically towards zero. It is also understood that the magnetic field in the ferrite will not enter the saturation domain because B1 is very weak at the level of L2.

Since, by definition, $\Phi 12 = M \times I1$, we can write: $M \approx \frac{B1 \times \mu r2 \times N2 \times S2}{I1}$

Since B1 at a distance X has been determined previously, it follows that:

$$M \approx \frac{\mu 0 \times \mu r1 \times \mu r2 \times N1 \times N2 \times \pi \times r2^2 \times r1^2}{2 \times X^3}$$

Starting from equation 4: $|U2(t)| \approx \frac{w0 \times M \times |E(t)|}{(R1+Rg)}$

and equation 5: $|I1| \approx \frac{|E(t)|}{(R1+Rg)}$, we deduce:

$$|U2(t)| \approx w0 \times M \times |I1| = \frac{w0 \times \mu 0 \times \mu r1 \times \mu r2 \times N1 \times N2 \times \pi \times r2^2 \times r1^2 \times |I1|}{2 \times X^3} \quad \text{(equation 6)}$$

Consider the magnetic moment μ1 (the ability of a magnetic circuit to align itself with an external magnetic field, such as a compass) of L1:

$\mu 1 = N1 \times |I1| \times \pi \times r1^2$, it follows that: $|U2(t)| \approx \frac{w0 \times \mu 0 \times \mu r1 \times \mu 1 \times \mu r2 \times N2 \times r2^2}{2 \times X^3}$

We can see that, for a given distance X and an inductance L2 (N2, r2, µr2), U2(t) depends on the magnetic moment µ1 of L1, w0, and µr1.

An example of minimum magnetic moment (12 A.m^2), specified for a caving device that must send a positioning signal through 80 m of rock, is given in [6] on page 4.

Experimental determination of M

Starting from equation 6 (assuming M and I2 to be very small), we have:

$$|U2(t)| \approx |e2(t)| = \left|\frac{d(\Phi 2)}{dt}\right| = w0 \times M \times |I1|$$

The mutual inductance M can therefore be determined experimentally using the expression $M \approx \frac{U2eff}{w0 \times I1eff}$ with w0 (=2 x π x f0) assumed to be known, while U2eff and I1eff can be measured (see §5.1, test no. 6).

4.5 Determination of the ideal solenoid L1

As indicated in §4.4, for U2(t) to be maximum with respect to the “generator” part, which includes solenoid L1, the magnetic moment µ1 ($\mu 1 = N1 \times |I1| \times \pi \times r1^2$) must be maximum, as well as µr1 (if ferrite is present) . I1 depends on the LF amplifier, so the more powerful it is (low Rg and high E), the higher I1 will be.

As for w0, increasing it will improve U2 linearly but, at the same time, will increase the value of R1 even more rapidly, which will decrease U2 (see §4.5.4). There is therefore a compromise to be made on the resonance frequency.

4.5.1 Assumption Rg<<R1 (i.e. a powerful BF amplifier is present at the output of PC No. 1's sound card)

µr1, N1, r1, and R1 depend directly on L1. Let us assume that Rg is close to 0 Ω and that R1 is much greater than Rg. It follows that, starting from equation 5, we can deduce that $|I1| \approx \frac{|E(t)|}{R1}$. So $|U2(t)| \approx \frac{w0 \times \mu 0 \times \mu r1 \times \mu r2 \times N1 \times N2 \times \pi \times r2^2 \times r1^2 \times |E(t)|}{2 \times X^3 \times R1}$

In this case, based on this expression of |U2(t)|, we see that to maximize U2 as a function of L1, we must maximize the expression $P = \frac{\mu r1 \times N1 \times r1^2}{R1}$. Let us assume that we have a copper wire of length Lf. We know that $Lf \approx 2 \times \pi \times r1 \times N1$

Furthermore, $R1 = \frac{\rho \times Lf}{Sf} = \frac{\rho \times 2 \times \pi \times r1 \times N1}{Sf}$, where ρ is the resistivity of the copper wire, Lf is its length, and Sf is its average cross-sectional area.

Therefore, $P = \frac{\mu r1 \times N1 \times Sf \times r1^2}{\rho \times 2 \times \pi \times r1 \times N1} = \frac{\mu r1 \times Sf \times r1}{\rho \times 2 \times \pi}$

We can see that the term N1 (number of turns of L1) has disappeared. It is therefore better to have the largest possible turn radius r1. But if r1 is large, ferrite cannot be inserted. The ideal solution would be to form a single very large turn with radius r1 = Lf / (2 x π) using the available wire and dispense with ferrite.

In practical terms, we will be limited on the radius r1, so we will still need to have a certain number of turns, because with a single turn we will no longer comply with the assumption Rg<<R1. We will space the turns apart from each other on a single layer preferably, to limit the proximity effect.

For this scenario, see also [6] page 6.

The cross-sectional area Sf should be as large as possible, which is easily understood because the larger Sf, the lower the resistance R1, and therefore the greater the current and primary magnetic field. Copper tubes can be used, given that the current propagates along the tube's surface. Refer to the skin effect in §4.5.4 and replace Sf with Su (§4.5.4). We must remain within the assumption Rg << R1; therefore, if R1 is very small, the generator must be extremely powerful (very low Rg).

4.5.2 Assumption Rg>>R1 (i.e. without LF amplifier)

Here, Rg is very large compared to R1, meaning that we do not have an audio amplifier, but, for example, only the headphone output of the sound card. Clearly, the performance will not be very good. The goal here is therefore to find the best possible compromise.

In the case of Rg>>R1, starting from equation 5, we deduce that $|I1| \approx \frac{|E(t)|}{Rg}$ with Rg fixed.

In this case, starting from this I1and on the equation 6, we see that the expression to be maximized is therefore: $P = \mu r1 \times N1 \times r1^2 = \frac{\mu r1 \times Lf \times r1^2}{2 \times \pi \times r1} = \frac{\mu r1 \times Lf \times r1}{2 \times \pi}$ with $N1 \approx \frac{Lf}{2 \times \pi \times r1}$

Here too, for a given length Lf (and the highest possible), it is better to have large coils. But since Rg>>R1, we can afford to have the largest possible product μr1×Lf×r1, with many large coils without ferrite. We can also opt for many small turns if we have ferrite with very high relative permeability (μr1).

4.5.3 Assumption that Rg is of the order of R1 (neither much higher nor much lower)

In this case, based on equations 5 and 6, the expression to be maximized is:
$P = \frac{\mu r1 \times N1 \times r1^2}{(R1+Rg)}$ with $N1 \approx \frac{Lf}{2 \times \pi \times r1}$ and $R1 = \frac{\rho \times Lf}{Sf}$ Therefore $P = \frac{\mu r1 \times Lf \times r1}{2 \times \pi \times (\frac{\rho \times Lf}{Sf} + Rg)}$

There is no simple solution here. The designer will have to test different hypotheses.

4.5.4 Theoretical and experimental determination of L1 and R1 (resistance of solenoid L1)

Practical and theoretical determination of L1

L1 is measured with an RLC meter and does not pose any particular problems. However, the inductance will not be measured at the frequency of use and there will therefore be a small error related to the distributed capacitance of the solenoid, whose reactance depends on the frequency.

Note that there are a few experimental formulas for calculating inductance, but they are not very accurate. Here is one, based on Figure 5:

$$L1(H) = \frac{\mu r \times 7.7E - 6 \times (r1 \times N1)^2}{2 \times r1 + 2.2 \times l1}$$

Theoretical determination of R1

Of course, R1 can be measured with an ohmmeter, but this measurement will not take into account the following losses, which always lead to an increase in resistance (and therefore unnecessary heat production) with frequency:

- proximity effects between turns and between layers,
- skin effect,
- hysteresis and saturation losses, if ferrite is used,
- eddy current losses, if ferrite is used.

The simple ohmic measurement of R1 is therefore optimistic.

Experimentally, we find that, to simplify, up to 4000 Hz, the increase in the value of R1 is linear with frequency (f0), but then tends to increase as $f0^{1.7}$. For example, for the ohmic resistance R1 of L1 equipped with ferrite, we measure 6.4 Ω continuously. This increases to 16.1 Ω at 2700 Hz, 27.4 Ω at 4200 Hz, and 336 Ω at 16000 Hz. Therefore, increasing the frequency too much can be counterproductive, since the loss due to R1 (at $f0^{1.7}$) can be greater than the linear gain due to w0. A better compromise on the frequency f0, ensuring maximum gain (i.e. maximum U2), must be made.

Proximity effects between adjacent turns and between layers

For explanations, refer to [7] pages 7 and 8 and the abacus in [7] page 8.

Skin effect

The skin thickness in m is $\delta=\sqrt{\frac{\rho}{\pi\times f\times\mu 0}}$ ([2] page 305) with $\rho=1.72\times10^{-8}$ Ω.m for copper.

With regard to the Joule effect, half the skin thickness must be considered (see [2] page 692), treating the tube as a thin plate.

If r is the radius of the wire cross-section, the effective cross-sectional area of the wire Su is equal to: $Su = (\pi \times r^2) - \left(\pi \times (r - \frac{\delta}{2})^2\right)$ if r>δ/2

or $Su = \pi \times r^2$ if r≤δ/2 (no skin effect)

For example, at 3200 Hz, the half-skin depth δ/2 is 0.583 mm (≈0.6 mm), so a wire with a minimum diameter of 1.2 mm should be suitable. In fact, in this case, even if r≤δ/2, there is a slight skin effect. See the chart in [7] on page 7.

Losses in ferrites

This is a complex subject. See [7] on page 8 and [8].

Experimental determination of R1

Rather than estimating R1 theoretically, a measurement can be taken.

For example, starting from Figure 2:

- a continuous sinusoidal signal is generated from an LF generator at the resonance frequency f0,
- U1 and I1 are measured,
- and R1 = U1 / I1 is deduced.

Note that this method is described in [6] pages 4 and 5.

4.6 Determination of the ideal solenoid L2

Starting from equation 6 in §4.4, for U2(t) to be maximum with respect to L2, we see that the product $P = \mu r2 \times N2 \times r2^2$ must be maximum. Since Rc>>R2, this is the same case as that described in §4.5.2 (Rg>>R1) and the conclusion is that we must have the largest possible product $\mu r2 \times Lf2 \times r2$, with Lf2 being the length of wire used for the L2 winding.
Here, the value of R2 is secondary because, being much lower than Rc, it will have very little effect on the value of U2.
As a result, there is no real limit to Lf2 (apart from the cost of the copper wire, the patience of the manufacturer, and compliance with the inequality Rc>>R2). In this context, very fine wire is sufficient.
We can also opt for many small turns if we have a ferrite with very high relative permeability (μr2).

> Note: if the input of the LF amplifier is an FET (and therefore has very high input impedance), Lf2 can be very large...

5. Testing and experimentation with near-field magnetic field transmission

5.1 Testing

For these tests, an LF generator (JDS6600) with internal resistance Rg=50 ohms and a digital AC voltmeter/ammeter (RACAL 4002), usable up to 10 kHz and with input impedance 1 MΩ/75 pF, were used. An RLC meter (M4070) was used to measure inductance and capacitance. A standard multimeter was used for resistance measurements.

> Note 1: Inductance values have a certain degree of uncertainty due to the fact that an inductor also has a certain distributed capacitance. Therefore, the inductance value displayed by the RLC meter depends on the frequency at which the measurement was made. It should therefore come as no surprise if the resonance frequency does not correspond exactly to the one calculated on the basis of this measurement (there is a small deviation).
> Note 2: The measuring instruments used have not been calibrated (as in the professional world), so the measurements are of “amateur” quality...

Referring to Figure 2:

- Coil L1 is a coil of wire purchased online (reference “ZDLSDL Lacquered copper wire - Coil of insulated copper wire – 0.6 mm - 100 m”). Its characteristics are as follows: 100 m of insulated copper wire with a diameter of 0.6 mm, with an average radius r1 of 14.75 mm (r1 min=10.5 mm and r1max=19 mm). Its inductance L1=13.2 mH (measured with an RLC meter) and its resistance R1 measured with an ohmmeter: 6.4 Ω. The length of the coil is 53 mm. There must be approximately 100 / (π x 14.75E-3 x 2)= 1079 turns.
  A ferrite element was inserted on the axis, bringing the inductance to 44.2 mH. The relative permeability μr1 is therefore equal to 44.2 / 13.2 = 3.35.
- Coil L2 is identical to L1.

A ferrite element was inserted on the axis, bringing the inductance to 71.3 mH. The relative permeability μr2 is therefore equal to 71.3 / 13.2 = 5.40.

Below is a screenshot of the test bench, measuring U2.

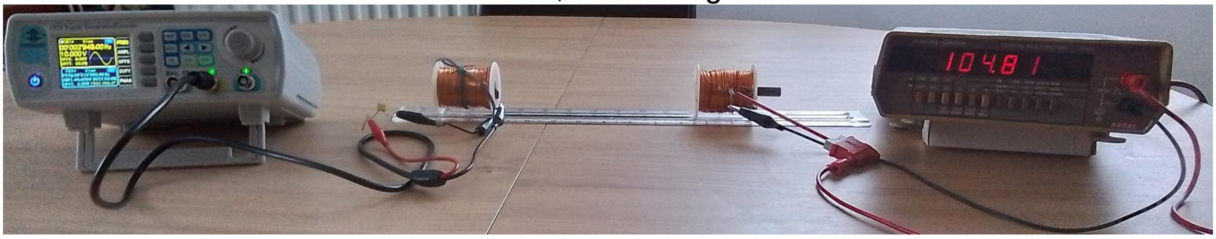

Figure 6: Test bench

Test No. 1: Determination of C1

In FT8, we are limited to 3000 Hz. We will therefore choose a resonance frequency f0 slightly below 3000 Hz. Starting from $C1 = \frac{1}{L1 \times (2 \times \pi \times f0)^2}$ and after a few adjustments, we find that a 68.9 nF capacitor gives a resonance frequency f0 of 2945 Hz. Note that to estimate the resonance frequency, we sweep the BF frequency range of the generator while measuring the current I1. The maximum current I1 (L1 and C1 compensating each other) corresponds to the resonance frequency.

Test No. 2: determining R1 as a function of frequency

R1 is measured using the experimental method described in §4.5.4.
At the resonance frequency of 2945 Hz, we find 20.8 Ω. Note that for a resonance frequency of 4200 Hz, we find R1=27.4 Ω, while at 16000 Hz we find 336 Ω. We can see that this real resistance increases rapidly with frequency, as explained in §4.5.4.
Remember that (L2 x w2) and R2 are very small compared to the input impedance of the "receiver" and are therefore not taken into account.

Another test was performed with L1 but without the ferrite. At a resonance frequency close to the previous one (2976 Hz), we find that R1 is equal to 10.0 Ω.
Therefore, the increase in R1 due to the ferrite is equal to 10.8 Ω, which is double the value of R1 without ferrite. This means that the presence of ferrite increases the inductance L1 and therefore the induced voltage U2 by a factor of μr1, but at the same time its presence reduces U2 by a factor of Kp:
Kp = (Rg+R1 without ferrite) / (Rg+R1 with ferrite) = 60.0/70.8=0.85.

Test No. 3: verification that U2(t) varies proportionally to μr1

At a given resonance frequency, by inserting or not inserting the ferrite into L1 and measuring the voltage U2 each time, it appears that the increase in U2 is proportional to approximately (0.83 x μr1). This factor of 0.83 is similar to the loss factor Kp calculated in test 2 (Kp=0.85) and can therefore be explained quite easily.

Test No. 4: verification that U2(t) varies proportionally to μr2

At a given resonance frequency, by inserting or not inserting the ferrite into L2 and measuring the voltage U2 each time, it appears that the increase in U2 is proportional to approximately (0.86 x μr2). This factor of 0.86 must correspond to the efficiency of the magnetic field concentration (see note in §4.4).

It should be noted, however, that at a large distance, the mutual inductance depends roughly on μr1 x μr2, which is very advantageous. Therefore, if ferrites are to be inserted into the solenoids, they should be chosen with the highest possible relative permeabilities μr1 and μr2.

Test No. 5: Verification that U2(t) varies proportionally to f0

If inductor L1 is not equipped with a ferrite core, R1 increases slowly (see test No. 2) and U2 is therefore practically proportional to f0. Losses begin to be noticeable at around 3000 Hz. For example, at 5460 Hz, the loss is 6%.

However, if L1 is equipped with a ferrite, R1 increases rapidly (see test 2 and §4.5.4) and losses therefore become significant with frequency. For example, they increase from 12% at 2940 Hz to 33% at 6190 Hz.

Note that the presence of a ferrite on L2 has no impact on this proportionality. However, losses due to R1 become much greater in the presence of a ferrite on L1.

To increase the range, it is therefore still advisable to increase the frequency to a value that allows maximum gain (§4.5.4). However, it must be taken into account that if f0 increases, the distance rmax in the near field (§2) decreases. For example, we will have:

- at f0=1000 Hz, rmax=300 km x 0.09=27 km,
- at f0=8000 Hz, rmax=37.5 km x 0.09=3.37 km.

Still using the same example, if at f0=1000 Hz, the maximum range is 100 m, at 8000 Hz it will be, all other things being equal, $100 \times \sqrt[3]{\frac{8000}{1000}}$ or 200 m (actually a little less due to greater losses on R1).

Note that a distance of 1 km seems to be the maximum for a link of this type.

Test No. 6: Experimental determination of M and comparison with the calculated value

As indicated in §4.4, the mutual inductance M calculated for a given distance X can be compared with the mutual inductance M measured using the expression $Mmeasured \approx \frac{U2eff}{w0 \times I1eff}$ (§4.4) with the angular frequency w0 assumed to be known, while U2eff and I1eff can be measured.

At f0=2943 Hz and X=25.8 cm, a test was performed and the following values were found: U2eff=104.45 mV and I1eff=50.42 mA, which gives Measured=0.112 mH.

The estimated value of M (§4.4)

$Mestimated \approx \frac{\mu 0 \times \mu r1 \times \mu r2 \times N1 \times N2 \times \pi \times r2^2 \times r1^2}{2 \times X^3}$ gives

Mestimated=0.115 mH

Mestimated is of the same order of magnitude (i.e. between half and double) as Mmeasured, which validates the method.

Test No.7: Verification that the calculated voltage U2 is of the same order of magnitude as the measured voltage U2

In §4.4, we give the approximate theoretical value of U2:
$U2eff \approx w0 \times M \times I1eff$, which in this case is U2eff≈0.107 V.
The calculated value of U2eff (107 mV) is of the same order of magnitude (i.e. between half and double) as the measured U2eff (104.45 mV), which validates the method.

Test No. 8: Transmission test through a wall
The author tested transmission via the magnetic field through the same 28 cm thick exterior wall that blocked the electric field.

The results were as follows:

- U2 through the wall: 40 mV. As the pointing accuracy was not ideal, this should be considered a minimum value.
- U2 through the air, at the same thickness as the wall: 44.5 mV

We can see that the wall has degraded the signal very little. The wall is therefore (almost) transparent to the magnetic field.

5.2 FT8 experiment
The author carried out the basic test bench described in Figure 2. The test bench consists of the following elements:

- PC No. 1 (Windows 7) runs on battery power and uses Multipsk V.4.50 software in FT8 mode, beaconing at a maximum frequency of 2700 Hz and transmitting a frame every 15 seconds.
  Note: this frequency of 2700 Hz falls exactly on a harmonic of 50 Hz, which will slightly degrade the signal quality.

- The headphone output of the PC's integrated sound card has an internal resistance of 66 Ω and a maximum effective voltage E of 1.12 V at 2700 Hz.

- The external sound card of PC No. 2 is a Behringer U-phoria UM2, whose sound card input has an internal resistance of approximately 25 kΩ. It is preceded by a Kemo M040N microphone amplifier with 48 kΩ input impedance and 50 Ω output impedance. Its gain is approximately 40 dB. There is no bandpass filter.

- PC No. 2 (Windows 10) runs on battery power and uses Multipsk V.4.50 software in FT8 for decoding.
  Note: WSJT decodes FT8 (and FT4) better (by 3 or 4 dB) than Multipsk, due to its very powerful algorithms. However, Multipsk does not require PC time synchronization for decoding and allows the exchange of non-standard messages to make a standard QSO.

It is difficult to calculate the maximum range in advance, as it depends on the noise component in the environment, but above all on the 50 Hz (60 Hz in certain countries) harmonics caused by the electrical network in the induced voltage, as well as the noise voltage of the sound card and the microphone amplifier, all of which are unknown. However, under fixed conditions, once the maximum range for a measured induced voltage has been determined, it is possible to predict the maximum range if a parameter is modified.

The inductors used are those described in §5.1. The frequency used is 2700 Hz using a capacitance C1 of 82.8 nF.

The author set a distance of 7.7 m between the two solenoids, the aim being to determine the minimum voltage U1 ensuring FT8 transmission. A minimum voltage U1 of 0.107 V is found.

From this, we could determine, for example, that if the voltage U1 were 10 V instead of 0.107 V, thanks to a powerful LF amplifier, the maximum distance would increase to: $7.7 \times \left(\frac{10}{0.107}\right)^{1/3} = 34.9\ m$

Below, in Figure 7, is the transmitter part of the experiment. From left to right, we see PC No. 1, capacitor C1, and solenoid L1/R1. Figure 8 shows a screenshot (in English) of Multipsk in FT8 beacon mode.

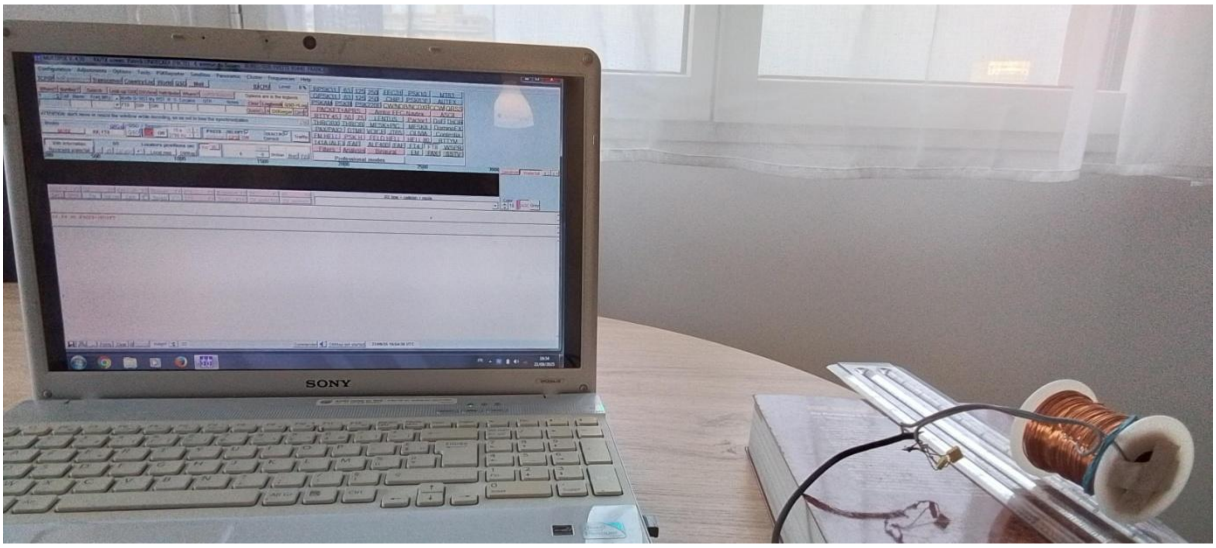
Figure 7: Transmitter part of the experiment

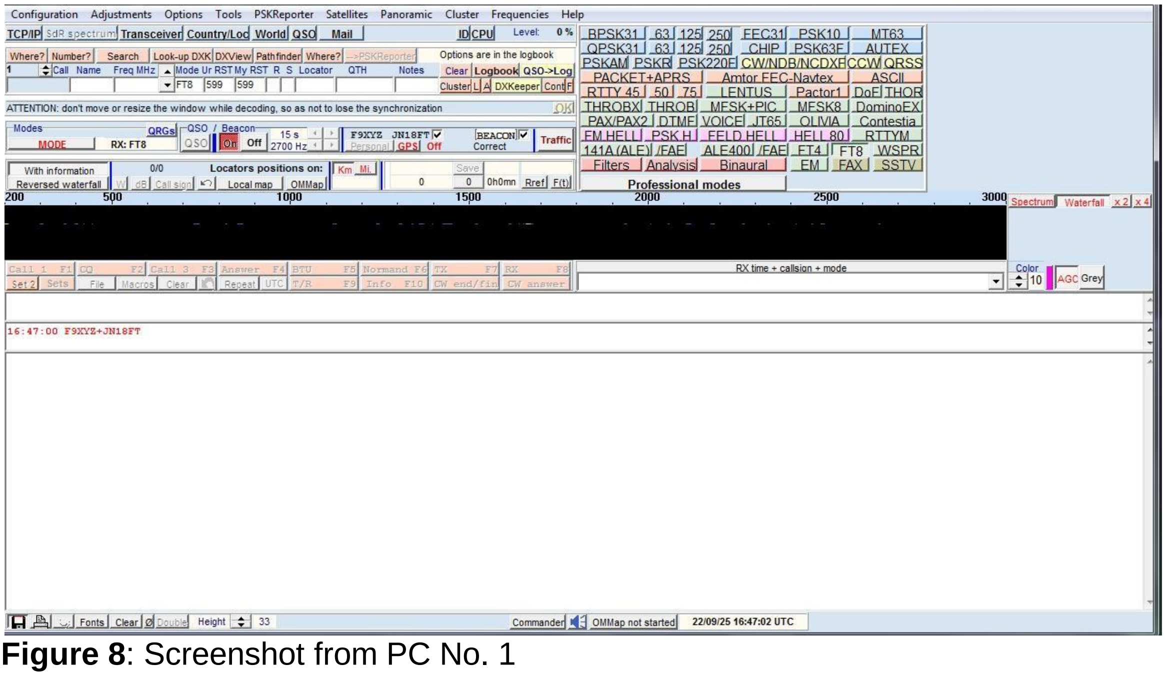

Figure 8: Screenshot from PC No. 1

Below, in Figure 9, is the receiver part of the experiment. From left to right, we see the L2/R2 solenoid, the microphone amplifier powered by a 9 V battery, the external sound card, and PC No. 2. Figure 10 shows a screenshot (in French) of Multipsk in FT8 decoding mode at 2700 Hz.

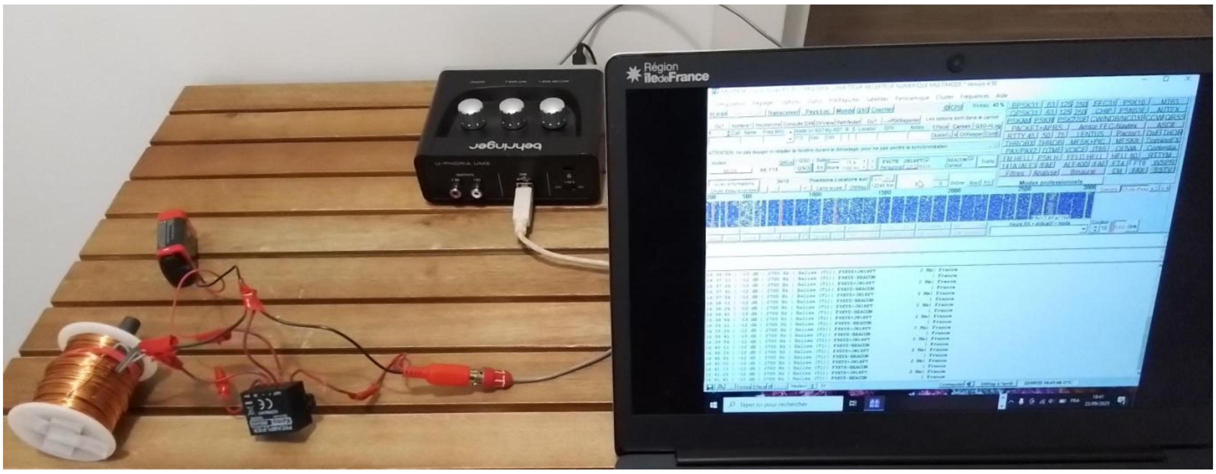

Figure 9: Receiver part of the experiment

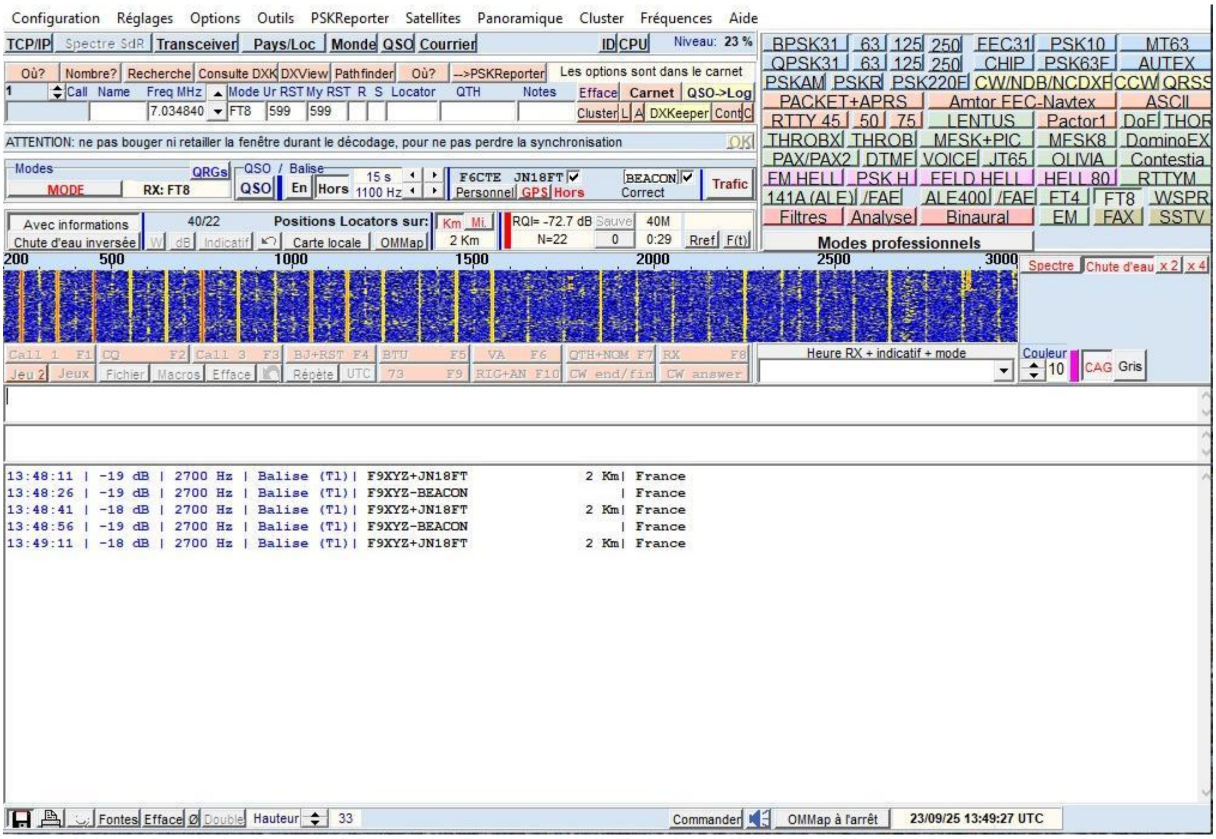

Figure 10: Screenshot from PC No. 2

As indicated above, the following possibilities for increasing the maximum distance are:

- inserting a powerful LF amplifier (§4.1) to increase E(t) and decrease Rg,
- maximizing the efficiency of solenoids L1 and L2 (§4.5.4 and §4.6),
- improve the characteristics with regard to noise, the sound card, and the "microphone" amplifier, and equip the amplifier with a narrow bandpass filter around the transmission frequency,
- to a certain extent, increase the frequency (§4.5, §4.5.4, and §5.1 tests 2 and 5).

Bear in mind that the gain G on the induced voltage U2 will result in an increase in range of only $\sqrt[3]{G}$. For example, a gain of 8 on U2 will result in an increase in range of 2.
Note that due to this rapid decrease following $1/X^3$ (X being the distance), it seems difficult to exceed a maximum range of 1 km.

It should also be noted that using the LF amateur radio frequency in the 137 KHz band would allow for a maximum range in the near field of approximately 200 m (which is low). Then, after transforming the magnetic field into an electromagnetic field, the received voltage will evolve according to 1/X, but the ability to pass through materials will be greatly reduced due to the electrical component inherent in this type of field.

6. Conclusion

We first showed that magnetic data transmission in the near field (§2) is better than transmission via the electric field (§3) if the induction has to pass through materials.
In sections §4.1 to §4.4, we reduced the physics of this type of transmission to relatively simple formulas.
In sections §4.5 and §4.6, we determined the best way to construct solenoids L1 and L2.
The tests in §5.1 validated the physical approach. Finally, an experiment was conducted in FT8 (§5.2).

While it is certain that this type of transmission is not intended for DX, it is still interesting to know the ability of the magnetic field to pass through materials. This quality, used in caving, could perhaps meet the needs of certain HAMs.

7. References

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Appendix A
Maxwell's equations under the AQSR Hypothesis

The following is taken from [2]. The four Maxwell's equations are presented in local form (i.e. over a small volume element, using differential operators) and in integral form.
The equation is given in steady-state (without variation of the parameters over time), then in variable state, and finally in variable state but in the near field (AQSR).

> Note : in North America, the vector operator « **rot** » (magnitude and axis of rotation) is noted « **curl** »

Maxwell-Faraday (circulation of the electrostatic field along a closed contour C)
In steady-state: $rot(\mathbf{E}) = 0$ or $\oint_C \mathbf{E}.d\mathbf{r} = 0$ (i.e. in this case, an electrostatic field line cannot close on itself).

In variable regime and AQSR: $rot(\mathbf{E}) + \frac{\partial \mathbf{B}}{\partial t} = 0$ or $\oint_C \mathbf{E}.d\mathbf{r} + \int_S \frac{\partial \mathbf{B}}{\partial t}.\mathbf{n}.dS = 0$ with the surface S resting on the contour C (or $e(t)(Volt) = -\frac{d\phi}{dt}$, called Faraday's law)

> Note: in other words, a changing magnetic field generates an electric current, as, for example, in the case of eddy currents.

Maxwell-Thomson (conservation of the magnetic field flux φ through a closed surface S bounding a given volume)

In steady-state, variable, and AQSR regimes: $div(\mathbf{B}) = 0$ or φ=$\oint_S \mathbf{B}.\mathbf{n}.dS = 0$

Maxwell-Gauss (determination of the electrostatic field flux ψ through a closed surface S bounding a given volume)

In steady-state and variable regimes: $div(\mathbf{E}) = \frac{\rho}{\varepsilon 0}$ or $\Psi = \frac{Q_{in}}{\varepsilon 0} = \oint_S \mathbf{E}.\mathbf{n}.dS$

ρ is the volume charge and Qin is the internal charge within the volume defined by the closed surface S.

In AQSR (but this is also true for the lower part of the electromagnetic spectrum): for a conductor, we have ρ=0 and therefore div(E)=0 and Qin=0. There are no charges inside the conductor, which is therefore neutral. The charges are located on the surface.

Maxwell-Ampère (circulation of the magnetic field along a closed contour C)

In steady state: $rot(\frac{\mathbf{B}}{\mu 0}) = \mathbf{J}$ or $\oint_C \mathbf{B}.d\mathbf{r} = \mu 0.I$ (i.e. the circulation of the magnetic field B along a closed contour C is proportional to the total intensity I of the current flowing through any surface resting on this closed contour C).

In variable state: $rot\left(\frac{\boldsymbol{B}}{\mu 0}\right) - \frac{\partial(\varepsilon 0.\boldsymbol{E})}{\partial t} = \boldsymbol{J}$ or $\oint_C (\frac{\mathbf{B}}{\mu 0}).d\mathbf{r} - \int_S \frac{\partial(\varepsilon 0.\mathbf{E})}{\partial t}.\mathbf{n}.dS = I$

In AQSR: $\int_S \frac{\partial(\varepsilon 0.\mathbf{E})}{\partial t}.\mathbf{n}.dS$ is very small for a conductor and can be neglected ([2] page 296). Note that $\frac{\partial(\varepsilon 0.\boldsymbol{E})}{\partial t}$ is the "displacement current".

We therefore have: $rot(\frac{\mathbf{B}}{\mu 0}) \approx \mathbf{J}$ or $\oint_C \mathbf{B}.d\mathbf{r} \approx \mu 0.I$ (as in steady state)

> Note: in other words, an electric current generates a magnetic field as, for example, in the case of an infinite solenoid ($B = \mu 0 \times n \times I$ with B the magnetic field on the axis and n the number of turns per m).

Other useful equations

Ohm's law is written: $\boldsymbol{J} = \gamma \times \boldsymbol{E}$ with γ the electrical conductivity.

The electric charge balance is written:
$div\,\boldsymbol{J} = -\frac{\partial \rho}{\partial t}$ or $I = \oint_S \boldsymbol{J} \times \boldsymbol{n} \times dS = -\frac{dQ}{dt}$, this is because the charge is a conservative quantity (like mass for example).